Trajectory Generation in Guided Spaces using NTG Algorithm and Artificial Neural Networks

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Problem

- Computation of **low-observable** trajectories in real-time with the NTG algorithm.
- Low-observable: Minimum **probability of detection** by radars.
- Formulated as a sequence of fixed final point problems with nonlinear dynamical constraints.
- Radars are considered as guiding the state space, rather than rigid obstacles.
- How to improve analytical expression of guidance?
Aircraft and Detection Models

[Diagram with boxes and arrows representing the flow of information from Target Waypoint, Velocity, Current Position, Heading to Aircraft Model, Aircraft Attitude, Coordinate Transform, Next Position, Radar Position, Elevation/Azimuth, Signature Lookup, Radar Signature, PD Lookup, Detection Probability, and Range.]
Problem Formulation

\[ \Phi_0 = W_t T \]
\[ \int_0^1 \left( \frac{W_u}{T^2} \left[ \left( \frac{dn_a}{d\tau} \right)^2 + \left( \frac{de_a}{d\tau} \right)^2 \right] - W_p \cdot p n d \right) T d\tau \]

Performance Index Components

Initial Constraints:
\[ n_a(0) \leq n_a(\tau)|_{\tau=0} \leq n_a(0) \]
\[ e_a(0) \leq e_a(\tau)|_{\tau=0} \leq e_a(0) \]
\[ \dot{n}_a(0)T \leq \frac{n_a(\tau)}{d\tau}|_{\tau=0} \leq \dot{n}_a(0)T \]
\[ \dot{e}_a(0)T \leq \frac{e_a(\tau)}{d\tau}|_{\tau=0} \leq \dot{e}_a(0)T \]

Velocity and Curvature Constraints:
\[ \frac{v_{lo}}{v_{hi}^2} \leq \frac{W_v}{T^2} \left( \left( \frac{dn_a}{d\tau} \right)^2 + \left( \frac{de_a}{d\tau} \right)^2 \right) \leq 1 \]
\[ -1 \leq W_c \frac{\frac{dn_a}{d\tau} \frac{d^2 e_a}{d\tau^2} - \frac{d e_a}{d\tau} \frac{d^2 n_a}{d\tau^2}}{\left( \frac{dn_a}{d\tau} \right)^2 + \left( \frac{de_a}{d\tau} \right)^2} \leq 1 \]

Final Constraints:
\[ 0 \leq W_f \left( (n_a(1) - n_f)^2 + (e_a(1) - e_f)^2 \right) \leq 1 \]
\[ \theta_{lo} \leq W_d \tan^{-1} \left( \frac{\frac{d e_a}{d\tau}}{\frac{d n_a}{d\tau}} \right) \leq \theta_{hi} \]
Nonlinear Trajectory Generation

System Dynamics:
\[ \dot{x} = f(x, u) \]

State and Input Constraints:
\[
\begin{align*}
    lb_0 & \leq \psi_0(x(t_0), u(t_0)) & \leq ub_0 \\
    lb_f & \leq \psi_f(x(t_f), u(t_f)) & \leq ub_f \\
    lb_i & \leq S(x, u) & \leq ub_i
\end{align*}
\]

Cost Function:
\[
J = \phi_0(x(t_0), u(t_0)) + \phi_f(x(t_f), u(t_f)) + \int_{t_0}^{t_f} L(x(t), u(t)) dt
\]

New Outputs:
\[
z = G(x, u, u^{(1)}, ..., u^{(r)}), \quad (x, u) = H(z, z^{(1)}, ..., z^{(s)})
\]

Using B-spline representation:
\[
z_1(t) = \sum_{i=1}^{p_1} B_{i,k_1}(t) C_i^1, ..., z_q(t) = \sum_{i=1}^{p_q} B_{i,k_q}(t) C_i^q
\]

New Cost Function:
\[
\min_{y \in \mathbb{R}^M} F(y) \quad \text{subject to} \quad lb \leq c(y) \leq ub
\]
\[
y = (C_1^1, ..., C_{p_1}^1, C_1^2, ..., C_{p_2}^2, ..., C_1^q, ..., C_{p_q}^q)
\]

- Based on a combination of nonlinear control theory, spline theory, and sequential quadratic programming
- Three key steps:
  1. Find a new set of outputs of a system using the differential flatness property
  2. Further represent these outputs in terms of B-splines
  3. Use NPSOL to solve the coefficients of the B-spline functions
Feedforward networks with sigmoidal units are \textit{universal approximators}.

Superior regression performance while avoiding over-fitting the sampled data.

Given I/O samples, parameters adjusted typically by gradient methods.
Connectionist Approximation

The considered approximator is a network of nonlinear nodes, each realizing the I/O relation:

\[ y = \phi \left( b + \sum_{i=1}^{n} w_i x_i \right) \]

These nodes are organized in a layered structure to implement

\[ \hat{p}(s, r) = W_3 \Theta \left( W_2 \Theta \left( W_1 \begin{bmatrix} s \\ r \end{bmatrix} + b_1 \right) + b_2 \right) + b_3 \]

Given a samples of a desired I/O relation, determining the optimal parameters is a matter of minimizing

\[ E(\mathcal{W}) = \sum_{i=1}^{s} \left\| t_i - \Phi(\mathcal{W}, x_i) \right\|_2^2 \]
### Signature and PD Lookup Tables

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**Large UAV Signature Data.**

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**Large UAV / Medium SAM Probability of Detection (Acquisition, Paq) Data.**
Approximation of Tabular Data

Less than $10^{-7}$ MSE error on the tabular data has been achieved with 3-layer networks of 8 (PD) and 11 (Signature) nonlinear nodes.
B-Spline Approximation
(same accuracy on tabular data)

Signature

Probability of Detection

Azimuth Angle (rad)  Elevation Angle (rad)

Probability of Detection (pd)

Range  Signature
Connectionist vs Spline Approximators

Connectionist approximation
8 / 11 Sigmoidal Nonlinear Nodes

Tensor-product B-spline approximator
Piecewise polynomials of order 4x2
Connectionist vs Spline Approximators

Connectionist approximation
8 / 11 Sigmoidal Nonlinear Nodes

Tensor-product B-spline approximator
Piecewise polynomials of order 4x2
**Connectionist vs Spline Approximators**

- **Connectionist approximation**
  - 8 / 11 Sigmoidal Nonlinear Nodes

- **Tensor-product B-spline approximator**
  - Piecewise polynomials of order 4x2

*B-Splines have a greater tendency to overfit the tabular data, even in lower orders.*
Conclusions and Future Work

- Accuracy in providing proper guidance to real-time NTG is essential.
- Connectionist tools offer smooth, accurate, and inexpensive approximation of guidance.
- It is possible to “prune” the architecture systematically without involving trial-and-error (unlike B-splines).
- B-Spline approximators are still inherent in the NTG procedure.