Performance Evaluation of RBF Network in Nearest Neighbor Classification

Mehmet K. Muezzinoglu

Computational Intelligence Lab.
University of Louisville
Louisville, KY, USA
Nearest Neighbor Classification

- Given a finite $M \subset \mathbb{R}^n$, the set of prototype patterns, and $x \in \mathbb{R}^n$, calculate

  $$f(x) = \arg \min_{y \in M} d(x, y)$$

- NN rule can be implemented easily on digital computers which need to perform several *comparisons*.

- Design of neural NN classifiers is a particular problem. There are many results and methods aiming to make specific finite-state recurrent models approximate $f(\cdot)$. (Associative memories)
The Gradient System

- Assign a center to each element of $M$. Assume Gaussian RBFs.
- Then we have the functional

$$E(x) = \exp(-\gamma^1 \|x - c^1\|^2) + \ldots + \exp(-\gamma^1 \|x - c^m\|^2).$$

- The gradient system has the form

$$\dot{x} = \nabla_x E(x)$$

$$= -2 \sum_{i=1}^{m} \gamma_i (x - c^i) \exp(-\gamma^i \|x - c^i\|^2)$$

$$= -x \cdot 2 \sum_{i=1}^{m} \gamma^i \exp(-\gamma^i \|x - c^i\|^2)$$

$$+ 2 \sum_{i=1}^{m} \gamma^i c^i \exp(-\gamma^i \|x - c^i\|^2).$$
Network Diagram

\[ \Phi_1(.) \]
\[ \Phi_2(.) \]
\[ \Phi_m(.) \]

\[ x_1 \]
\[ x_2 \]
\[ \vdots \]
\[ x_n \]

\[ -c^1_1 \]
\[ -c^2_2 \]
\[ -c^m_m \]

\[ \gamma^1 \]
\[ \gamma^2 \]
\[ \gamma^m \]

\[ x_1 \]
\[ x_2 \]
\[ \vdots \]
\[ x_n \]
Neurodynamical Tools for Exact NN Classification

- Consider the RBF network

- A gradient system maximizing the energy will map any initial condition to the nearest center.
Problem

Given \( \ell \) Gaussians centered at \( c_1, \ldots, c_\ell \), all with equal variances, how many local maxima does their sum possess? How far are they located from the centers?

\[
E(\cdot) : \mathbb{R}^n \to \mathbb{R}, \quad E(x) = \sum_{i=1}^{\ell} \exp \left( -\gamma \|x - c_i\|_2^2 \right)
\]
Classifier Performance (cont’d)

Partitions realized by the proposed system for three $\gamma$ values (13.2712, 22.2702, 32.2712):
Character Recognition (cont’d)

20% distorted version (obtained by bit-inversion) of a sample text:

```
quick brown fox jumps over the lazy dog
```

Recognized text:

```
quick brown fox jumps over the lazy dog
```

Analog processing time $\approx 1\,ms$ for $K = 1000$. 
Image Reconstruction (cont’d)

Reconstructions from 40% salt-and-pepper noise.

Analog processing time $\approx 10s$ for $K = 10^8$. 
Questions?