

# Black generation using lightness scaling

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## ABSTRACT

This paper describes a method for constructing a lookup table relating a three-dimensional CMY coordinate system to CMYK colorant amounts in a way that maximizes the utilization of the printer gamut volume. The method is based on an assumption, satisfied by most printers, that adding a black colorant to any combination of CMY colorants does not result in a color with more chroma. Therefore the CMYK gamut can be obtained from the CMY gamut by expanding it towards lower lightness values. Use of black colorant on the gray axis is enforced by modifying the initial distribution of CMY points through an approximate black generation transform. Lightness values of a resulting set of points in CIELAB space are scaled to fill the four-color gamut volume. The output CMYK values corresponding to the modified CIELAB colors are found by inverting a printer model. This last step determines a specific black use rate which can depend on the region of the color space.

**Keywords:** color conversion, black replacement, four-color printing

## 1. INTRODUCTION

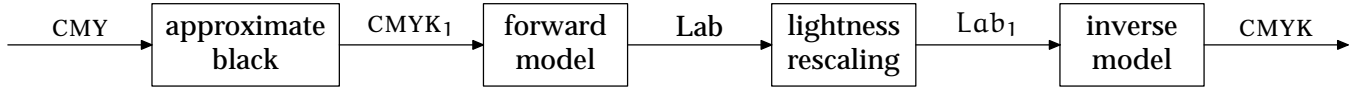
Satisfactory reproduction of colors requires the use of three colorants such as cyan, magenta, and yellow (CMY). Most printing systems augment this set with an additional black colorant (K). The advantages of using a CMYK colorant set include increasing the gamut size, reducing the costs of printing by saving consumption of expensive color inks, improving the detail rendition, and making gray balance more stable. However, the trichromatic nature of color vision permits any possible colors to be described by just three numbers, for example, by its CIELAB coordinates. The black generation method described here provides another three-dimensional coordinate system which covers the entire printer CMYK gamut. We choose to use the name CMY for the input coordinates though they no longer directly correspond to printer colorant amounts.

Many authors have described methods for converting from three to four colorant spaces. One popular solution is based on a combination of Under Color Removal (UCR) and Black Generation (BG) functions<sup>1</sup>:

$$\begin{aligned} PK &= \min(C, M, Y) \\ C_1 &= C - \text{UCR}(PK) \\ M_1 &= M - \text{UCR}(PK) \\ Y_1 &= Y - \text{UCR}(PK) \\ K_1 &= \text{BG}(PK) \end{aligned} \tag{1}$$

where  $(C, M, Y)$  are input values and  $(C_1, M_1, Y_1, K_1)$  are the outputs. The black generation function computes the black colorant amount based on the process black amount PK. The CMY values are reduced by an amount dependent on PK. Such a simple transformation does not preserve colorimetric relations and does not allow use of the whole printer gamut.

Approaches to black colorant calculation can be classified depending on the resulting range of reproducible colors. Methods include three color region, maximum black region, and four colorant region. Three color region techniques, sometimes called Gray Component Replacement (GCR), limits the resulting gamut to colors that can be printed with CMY colorants only. Nakamura and Sayanagi<sup>2</sup> described a solution of the first type based on Neugebauer equations. For GCR, the black level proportional to the minimum of the three input colorant values is first determined, then the printer model is inverted to find CMY values that match the original color produced without black. Kang<sup>3</sup> proposed a similar method using a spectral extension of the Neugebauer model which, for each device CMY value, finds a CMYK value with the same color. Maximum black region methods fix one of the



**Figure 1.** Block diagram of the black generation method.

resulting C, M, or Y values at zero, so that each color is reproduced by black and at most two chromatic colorants. The four colorant region techniques allow all four colorant amounts to be non-zero and result in the largest gamut.

Another important application of black generation techniques is for conversion from a device-independent space such as CIELAB to the device-dependent CMYK colorant space. For example, Hung<sup>4</sup> described a method for colorimetric black specification. In Hung's method, the maximum and minimum amounts of black with which each color can be reproduced are determined, then the CMYK colorant values with the desired black constrained to a value between these two numbers are found by inverting a printer model. The method described in this paper uses this approach as the last processing step.

## 2. METHOD DESCRIPTION

The black generation using lightness scaling method creates a lookup table relating CMY values to CMYK amounts, while expanding the color gamut addressable from that available with CMY colorants to the larger gamut volume possible using the CMYK colorant set. This method is based on an assumption, satisfied by most printers, that adding a black colorant to any combination of CMY colorants will not result in a more chromatic color and therefore the CMYK gamut can be obtained from the CMY gamut by expanding it towards lower lightness values.

The block diagram of our method is shown in Figure 1. We start with an initial distribution of points in a CMY space, for example, a regular cube of CMY values.

### 2.1. Approximate Black

An approximate conversion to CMYK is performed. We impose a restriction on this initial transformation that no output values are to exceed the original CMY gamut chroma boundaries. Another requirement is that color values corresponding to the upper surface of the printer gamut should not be modified, i.e.:

$$C_1 = C, \quad M_1 = M, \quad Y_1 = Y, \quad K_1 = 0 \quad \text{if } \min(C, M, Y) = 0 \text{ and } K = 0.$$

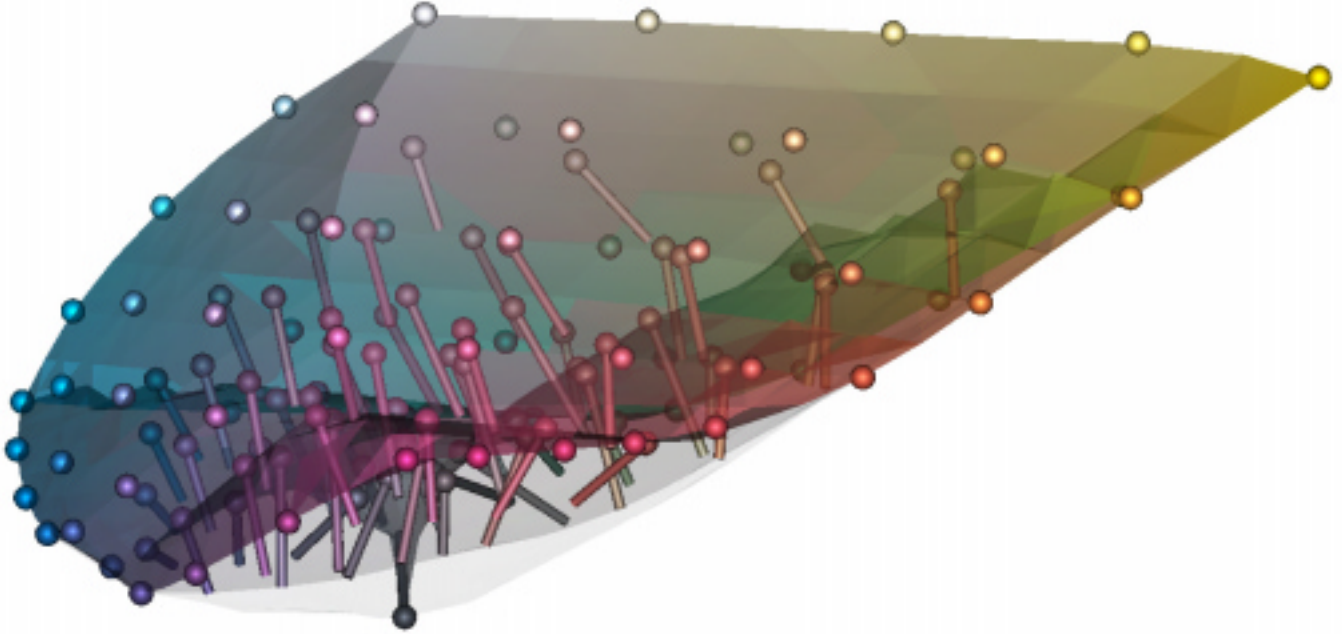
The simplest case of such a conversion is accomplished by just adding a  $K = 0$  component. However, sometimes we also want to impose a condition that balanced CMY values should be converted to pure black:

$$C_1 = M_1 = Y_1 = 0, \quad K_1 = C \quad \text{if } C = M = Y.$$

All the above conditions are satisfied by a simple black generation method known as achromatic reproduction, which is obtained from Equation (1) when  $BG(x) = x$  and  $UCR(x) = x$ :

$$\begin{aligned} K_1 &= \min(C, M, Y) \\ C_1 &= C - K_1 \\ M_1 &= M - K_1 \\ Y_1 &= Y - K_1. \end{aligned}$$

The final  $a^*$  and  $b^*$  coordinates of colors on ramps from black to primary and secondary colors are also implicitly defined in this step. By using nonlinear modifications of the above formula, desired colorant behavior can be obtained. A visualization of the color transformation corresponding to achromatic reproduction for a Lexmark Optra Color 45 inkjet printer is shown in Figure 2. The gamut boundaries in this figure were computed using alpha shapes.<sup>6</sup>



**Figure 2.** Visualization of the initial black generation step in CIELAB space: the largest shape is the full CMYK printer gamut, the medium one is the CMY gamut, and the smallest one is the CMYK gamut after achromatic reproduction transformation. Lines connect the original CMY cube values to modified CMYK values (marked with spheres).

## 2.2. Forward Model

CIELAB values corresponding to the CMYK values, obtained in the preceding step, can be obtained by printing a set of patches on the printer and measuring them or by developing a printer model. The second method is much more flexible and avoids remeasurements if the approximate black function needs to be changed. A number of different techniques can be used for implementing a printer model such as Neugebauer equations, lookup tables with interpolation, neural networks, fuzzy logic, or polynomial regression.<sup>3</sup>

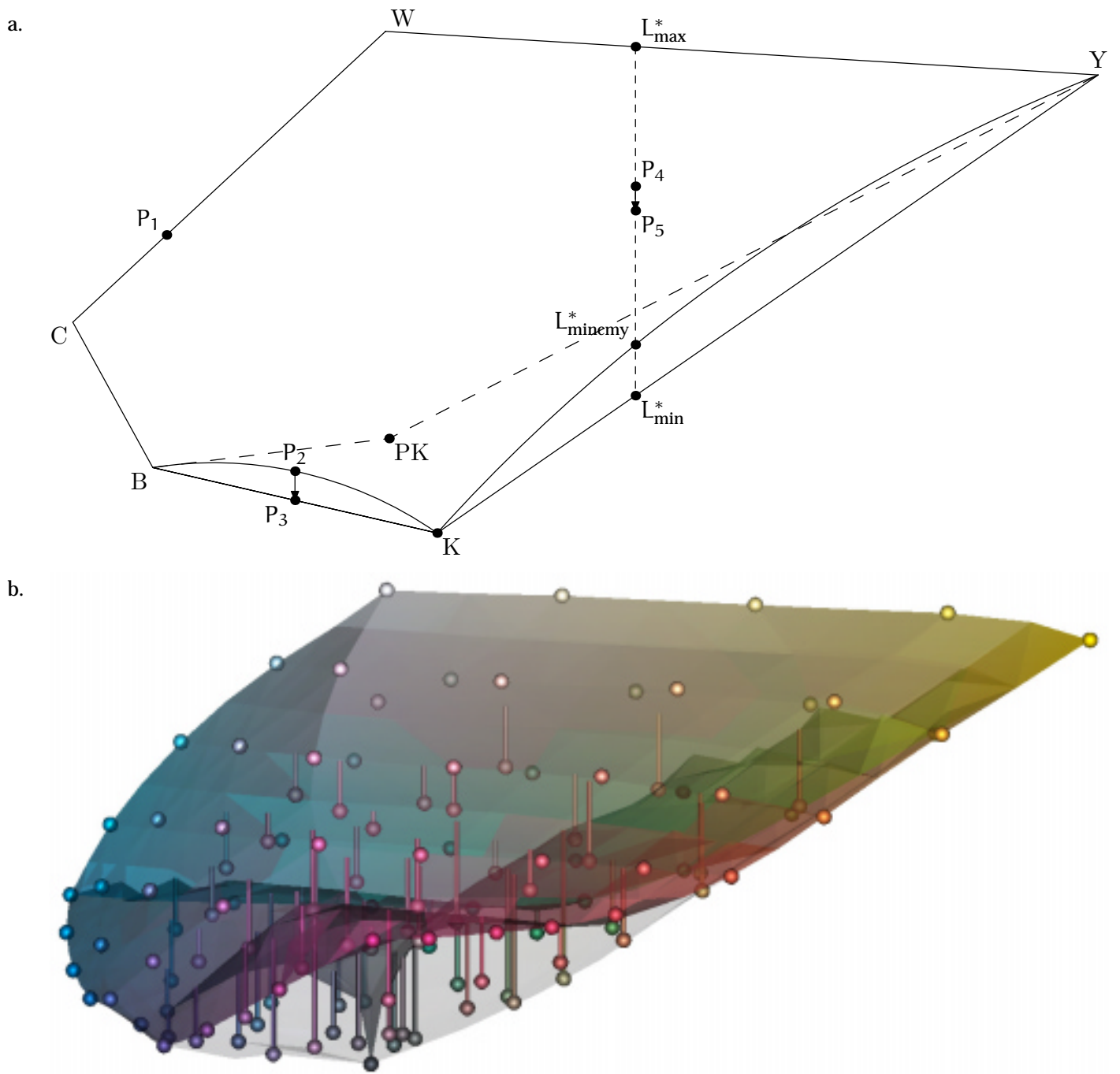
## 2.3. Lightness Rescaling

The CIELAB point distribution is modified by changing the lightness components in such a way that the upper surfaces of the CMY gamut are preserved and the lower surfaces are mapped to the bottom surface of the full CMYK gamut. For each point obtained in the previous step, we find three corresponding points with identical  $a^*$  and  $b^*$  coordinates. One of them is located on the top surface of the CMYK gamut which, according to our definition of the initial black generation step, is identical to the top surface of the CMY gamut. We denote the lightness of this point by  $L_{\max}^*$ . The other two points are located on the bottom surfaces of the CMY and CMYK gamuts and have lightnesses  $L_{\min\text{cmly}}^*$  and  $L_{\min}^*$ , respectively. In case that black is not the darkest point in the gamut, the values of  $L_{\min}^*$  can be clipped to be not less than  $L^*$  of black to avoid non-monotonic behavior in some regions of the transform. The modified  $L^*$  values are found by expanding the lightness range, in the simplest case by linear rescaling:

$$L_1^* = \begin{cases} L_{\min}^* + (L^* - L_{\min\text{cmly}}^*)(L_{\max}^* - L_{\min}^*) / (L_{\max}^* - L_{\min\text{cmly}}^*) & \text{if } L_{\max}^* - L_{\min\text{cmly}}^* \neq 0 \\ L^* & \text{otherwise.} \end{cases}$$

A diagram and a visualization of the lightness rescaling process is shown in Figure 3.

We developed two methods for determining the  $L_{\min}^*$ ,  $L_{\max}^*$  and  $L_{\min\text{cmly}}^*$  values: geometric and constrained optimization. These methods do not make any assumptions about the way CMY sub-gamuts for different values of K overlap and work in the presence of natural gamut boundaries.<sup>5</sup>



**Figure 3.** A diagram and a visualization of the lightness rescaling process in the CIELAB space. a. The dashed outline shows the bottom of the CMY gamut, the intermediate outline is the gamut after approximate black generation, and the outermost outline is the full CMYK gamut. The vertical dashed line has constant  $a^*$  and  $b^*$  coordinates. Point  $P_1$  is not modified because it is on the upper surface of the gamut. Point  $P_2$  is mapped from the bottom of the modified CMY gamut to a point  $P_3$  on the bottom of the CMYK gamut. Point  $P_4$  is inside the CMY gamut and is transformed to a point  $P_5$  which has proportionally rescaled lightness. b. The larger, transparent shape represents the full CMYK printer gamut, and the smaller one is CMYK after achromatic reproduction transformation. Lines connect the modified CMYK values to values after lightness scaling (marked with spheres).

The geometric method uses alpha shapes<sup>6</sup> to determine the description of the gamut boundaries in the form of surfaces composed of triangles. The intersections of the lines of constant  $(a^*, b^*)$  with the top surface of the CMYK gamut and the bottom surfaces of CMYK and CMY gamuts are found. A search for triangles which are intersected by this line is performed after the  $L^*$  coordinates are ignored by projecting all the triangles onto the  $a^*b^*$  plane. Figure 4 shows a such projection for the Lexmark Optra Color 45 inkjet printer gamut surface. Planar barycentric coordinates of a point  $(a^*, b^*)$  with respect to a triangle  $P_1P_2P_3$  where  $P_i = (L_i^*, a_i^*, b_i^*)$  are given by:

$$\begin{aligned} u &= [(a_2^* - a^*)(b_3^* - b^*) - (a_3^* - a^*)(b_2^* - b^*)]/A \\ v &= [(a_3^* - a^*)(b_1^* - b^*) - (a_1^* - a^*)(b_3^* - b^*)]/A \\ w &= [(a_1^* - a^*)(b_2^* - b^*) - (a_2^* - a^*)(b_1^* - b^*)]/A \end{aligned}$$

where  $A$  is the area of the triangle under consideration:

$$A = (a_2^* - a_1^*)(b_3^* - b_1^*) - (a_3^* - a_1^*)(b_2^* - b_1^*).$$

The line of constant  $(a^*, b^*)$  intersects a triangle if each of the coordinates  $(u, v, w)$  is greater than or equal to zero. Assuming that the gamut shape is sufficiently regular, for each  $(a^*, b^*)$  within the gamut chroma limits there will be exactly two triangles like that: one on the top and one on the bottom surface of the gamut. The  $L^*$  value for each of the intersection points is then computed from the weighted sum:

$$L^* = uL_1 + vL_2 + wL_3.$$

Most printing processes impose a limit on the Total Area Coverage (TAC), defined as the sum of the fractional areas covered by the halftone dots in the four separations. We can prevent exceeding the TAC limit by using only the points with colorant amounts less than TAC for computation of gamut surfaces.

The constrained optimization method<sup>7</sup> is adapted for the purpose of finding the maximum and minimum  $L^*$  values on the gamut surface by searching for the point in the gamut which is the closest to the point outside the gamut which has the same  $a^*$  and  $b^*$  coordinates. The color space error function is constructed by weighting the  $a^*$  and  $b^*$  (or chroma and hue) much more strongly than lightness, for example, by setting  $S_C = S_H = 1$  and  $S_L = 0.001$ . This produces the type of gamut mapping which transforms out-of-gamut points approximately vertically to points on the gamut surface. Lightness values range is limited by the TAC constraint incorporated in the optimization process.

## 2.4. Inverse Model

Finally, the expanded CIELAB values are inverted subject to specific constraints on the desired relationship between  $K$  and  $CMY$  and on the Total Area Coverage (TAC) limit.<sup>7</sup> This allows the black use rate to depend on the region of the color space. These  $CMYK$  values, corresponding to input  $CMY$  values, can be smoothed.

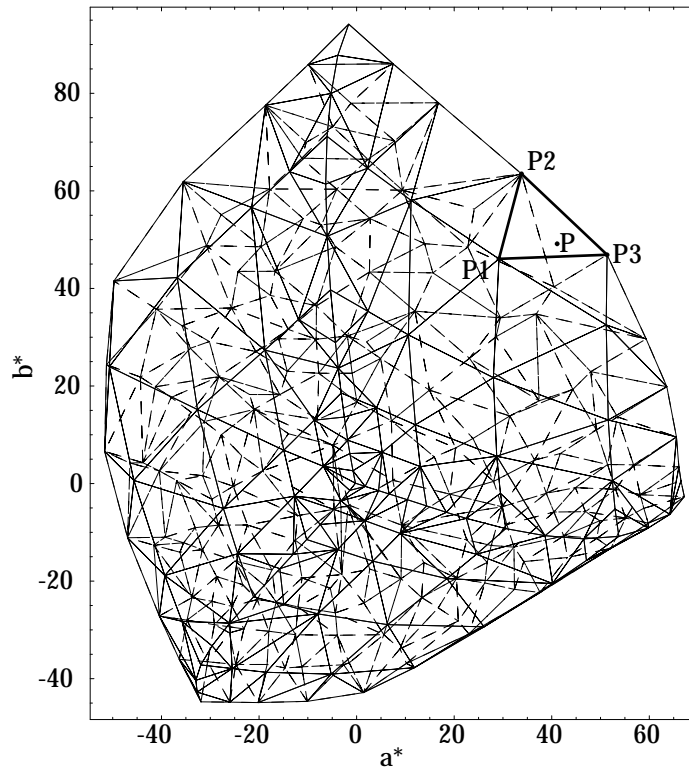
One useful smoothing kernel is based on a Laplace equation,<sup>8</sup> which in the one-dimensional case (for the edges) is:

$$u_j = \frac{1}{2}(u_{j-1} + u_{j+1}) \quad \text{for } j = 1, \dots, J-1$$

where  $J$  is the number of grid points. First,  $CMYK$  values for edges of the input  $CMY$  cube are smoothed while keeping the corner values fixed, then cube faces are smoothed while keeping the edges fixed, and finally the whole cube volume is smoothed. Alternatively, smoothing can be performed by fitting a global model to the set of values and then using it to obtain new values.

## 3. CONCLUSIONS

Our method for converting from a virtual three-colorant to a four-colorant space allows access to all colors from the printer gamut for which the TAC limits are not exceeded. This transformation can be interpreted as equivalent to distorting a unit cube so that the cube faces follow the gamut surface and its corners coincide with white, six primary and secondary device colors, and black. Another interpretation of our method is that it provides a smooth three-dimensional curvilinear coordinate system embedded in the CIELAB space.



**Figure 4.** Projection of the Lexmark Optra Color 45 inkjet printer gamut boundary wireframe on the  $a^*b^*$  plane. The solid and dashed lines correspond to the top and bottom gamut surfaces, respectively.

The main application of this method is to simulate a device RGB space for four-color printers. This can be helpful for partitioning the color management into the inverse mapping from a colorimetric space to a device-dependent three-dimensional space without concern for redundant colorants. A possible extension for adding colorants other than black is to perform the scaling not in the lightness dimension but towards the new colorant.

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