Gamut boundary determination using alpha-shapes

Tomasz J. Cholewo and Shaun Love
Lexmark International, Inc.
Lexington, KY, USA

Abstract

This paper proposes a solution to the problem of finding the boundary of the gamut of a color printing device or of a color image. A surface triangulation of a set of points in the color space is computed using an alpha-shape, which is a generalization of a convex hull applicable also to non-convex solids. The desired level of detail can be controlled by means of an alpha parameter. A method for selecting the suitable value of this parameter is proposed.

Keywords: color gamut, visualization, convex hull, alpha-shape

Introduction

A color gamut is a delimited region in color space, containing colors that are physically realizable by a given device or that are present in a given image. Knowledge of the color gamut surface is useful for many color science-related tasks such as visualization, gamut volume calculation, and deciding how colors outside the color gamut should be reproduced.

Approaches to reconstruction of the gamut surface can be divided into two groups: colorant space methods, which use the information about the connectivity in a device color space; and geometric methods, which are based only on a set of point coordinates in a device-independent (or colorimetric) color space such as CIELAB or CIECAM97s.

The colorant space methods are based on an assumption that a color space point lies on a surface of the gamut when at least one of the colorant coordinates attains its minimum or maximum value (Rolleston, 1993). Such identified surface points can then be connected to form a mesh describing the whole surface of the gamut. The resulting boundaries are called physical boundaries. When there are more than three colorants involved, this usually involves computing the gamuts of the three-colorant subprocesses and then finding their union.

In a method described by Braun and Fairchild (1996) the surface points identified in the colorant space are converted to the cylindrical CIELAB coordinates and projected on the L*h* plane. The points were triangulated using neighborhood information from the colorant space. The obtained gamut surface is represented by a matrix specifying the maximum chroma value attainable for given lightness and hue. This technique assumes that for each point on the gamut boundary there is at most one chroma value for a given combination, which is true for most typical printer gamuts but is not satisfied by some image gamuts. The corners and edges of the gamut may not be represented accurately because of the discrete location of the grid points.

Other authors used the characterization data (pairs of corresponding colorant space and color space values) to create a device model which then is employed for derivation of the gamut surface. This requires making additional assumptions about the behavior of the device and hence depends on the physics of the printing process, but in return provides a method for removing some measurement noise from the data and a possibility to create analytical representations of the gamut.

Mahy (1999) used the device characterization data to build a number of localized three-dimensional Neugebauer models. These models are analytically inverted to produce a set of closed contours situated on planes corresponding to constant lightness values in the color space. The color gamut contours of an n-colorant process for specific surfaces are determined as envelopes of contours of all its three-ink subprocesses.

Herzog (1998) proposed to represent the gamut by an analytical function based on a distorted cube. This provides an easy method of calculating the maximum chroma for any combination of lightness and hue but limits applicability of the method to cube-shaped gamuts.

As an alternative to colorant space methods, the geometric approaches work for any number of colorants and without knowledge of the colorant space data or the device model. Therefore they can be used for construction of gamut surfaces for arbitrary data sets such as measurements of targets with unknown underlying colorant specifications or for the set of colors present in an image. Similar to colorant space methods, many color space points are needed to describe the gamut surface precisely.

Mahy (1998) observed that for some printing processes certain colorant combinations result in colors that fall outside the region delimited by physical gamut boundaries. This phenomenon is caused by the presence of regions of non-monotonic mapping between the colorant and the color spaces which give rise to so-called natural boundaries corresponding to the local extrema of these mappings. The necessary condition for this to occur is to have some colorant combination inside the gamut produce the same color as some colorant combination from the physical boundary. The problem of natural boundaries which is very difficult to solve using colorant space methods can be addressed by geometric techniques by ensuring that the used color point set covers also the regions inside the colorant hypercube.

One simple geometric approach is to use a convex hull of the data set as the gamut surface. Unfortunately, in practice, non-convex (concave) surfaces are common in device gamut boundaries, and the convexity assumption usually leads to an overestimation of the gamut volume.

Balasubramanian and Dalal (1997) attempted to fix this deficiency by "inflating" the data set before computing its convex hull in such a way that the concave surfaces become convex for the purpose of generating the mesh. The disadvantage of this approach is the heuristic character of the method requiring precise selection of the center point and three parameters. This limits the method's applicability to printer-like gamuts. Over-inflation of the gamut may result in interior points being identified as surface points.

In this paper we present a new geometric method based on the alpha-shape of the set of points.

Alpha shapes

The concept of alpha-shapes developed by Edelsbrunner and Mücke (1994) formalizes the intuitive notion of "shape" for spatial point sets. The alpha-shape is a mathematically well-defined generalization of the convex hull and is a subgraph of the Delaunay triangulation. Given a finite point set, a family of shapes can be derived from the Delaunay triangulation of the point set; a real parameter α controls the desired level of detail. The set of all real alpha values leads to a whole family of shapes capturing the intuitive notion of "crude" versus "fine" shapes of a point set. Alpha-shapes have been used in scientific computing and engineering for a variety of purposes, such as molecular modeling, modeling and examination of tissue features, CAD/CAM solid surface reconstruction, mesh generation, and three-dimensional morphing between two shapes.

In three-dimensions the alpha-shape consists of many, possibly disjoint, simplices, i.e., tetrahedra, triangles, edges, and points. For $\alpha = 0$, the alpha-shape is identical to the original set of points S, and for $\alpha = \infty$, the set of all triangles in an alpha-shape is equal to the convex hull of S. A simplex belongs to the alpha-shape of S when there exists a sphere of radius α which does not contain any points of S and which has the property that all vertices of this simplex lie on its boundary.

For our purposes we are using only tetrahedra and triangles of the alpha-complex. Tetrahedra are used for volume computation. For gamut surface definition we use a subset of an alpha-shape consisting of all regular triangles, i.e., triangles which bound some tetrahedra which are also part of an alpha-shape, which we denote as an alpha-surface.

In practice the computation of the alpha-surface starts with finding the Delaunay triangulation of the point set S, which is a triangulation of the points in S such that no tetrahedron contains a point of S in its circumsphere. This results in a set of tetrahedra and convex hull triangles. Each face (triangle) is shared by exactly two tetrahedra or is a convex hull triangle bounding one tetrahedron. A triangle belongs to the alpha-surface when the value of the parameter α is between the radii of the smallest circumspheres for neighboring tetrahedra.

These concepts are illustrated for a two-dimensional case in Figure 1. An interactive demonstration is available on the Internet (Bélair, 1997).

Sorted radii of the smallest circumspheres of all tetrahedra form a so called α-spectrum. As the parameter α is increased, the alpha-shape changes only at values belonging to the α-spectrum, and therefore it is possible to create an ordered collection of alpha-shapes. In such a collection, alpha-rank denotes an index of a specific alpha-shape.

An important issue when using alpha-surfaces is selection of an appropriate value for the α parameter. When alpha-shapes are applied to the device gamut surface reconstruction we can make additional assumptions that make this task easier. The minimal useful value of α, which we will denote by α^*, is provided by the condition that all tetrahedra enclosed by the alpha-surface must belong to the alpha-shape. This corresponds to the intuitive notion that the gamut should not have any voids (regions in a shape that cannot be accessed from the outside).

Another requirement could be for the alpha-surface to consist of a single connected component. Since all triangles in an alpha-surface are regular we can determine the number of connected tetrahedral components by checking connectivity of a graph formed by the edges of the alpha-shape.

The alpha-surface obtained for α^* will sometimes be unnecessarily ragged as a result of not including tetrahedra located close to the surface. On the other hand, overly large parameter values will result in overestimation of the gamut volume by hiding some surface concavities. Therefore, the optimum value of α, especially for irregular image gamuts, is best determined experimentally, preferably using interactive visualization tools.

Results

We developed a visualization package in VRML 2.0 and JavaScript which allows us to interactively select the value

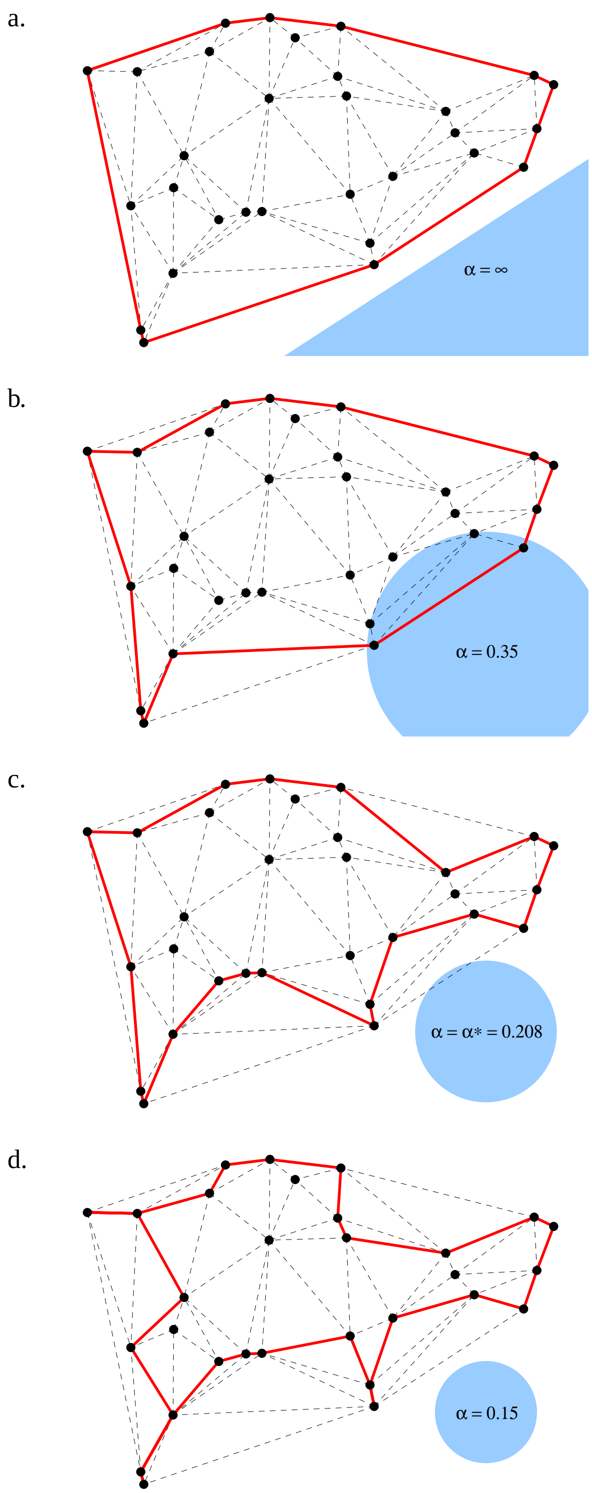

Figure 1: *A family of two-dimensional alpha-shapes for different values of α. Dashed lines show the Delaunay triangulation of the set of points and a thick line is the edge component of its alpha-shape. The disk in the right corner has radius α. The first alpha-shape ($\alpha = \infty$) is identical to convex hull and the disk is a half-plane.*

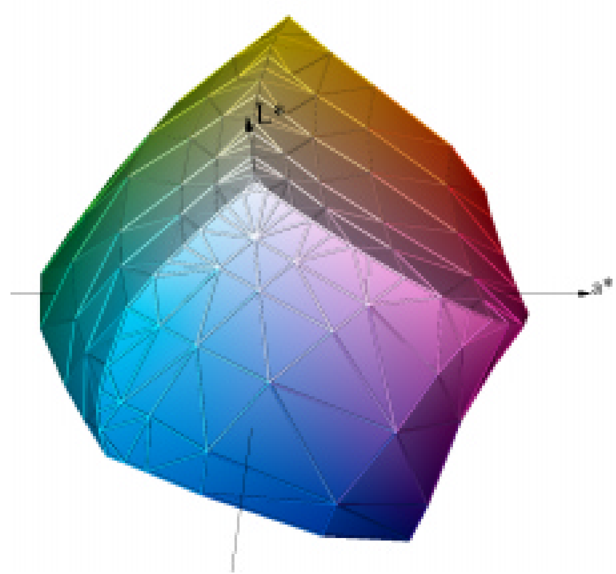

Figure 2: *Alpha-surface of the Lexmark Optra Color 45 inkjet printer based on IT8.7/3 chart measurements.*

of α and display the resulting alpha shape. Figure 2 shows an example of the VRML visualization of the gamut of an inkjet printer. Our user interface limits the allowed values of α to a range from α^* to infinity. This allows us to store only the triangles for which the upper limit of visibility is larger than α^*. A related software application created by the authors of the alpha-shape concept is publicly available from National Center for Supercomputing Applications (1996).

Figure 3 presents several signatures of the alpha-shape of a laser printer, that is, plots of some scalar values characterizing the family of alpha-shapes in function of their rank. These plots are useful in selecting the most appropriate value of α. We can observe that small changes in the α parameter above the α^* threshold cause only gradual changes in the volume and therefore are not critical for the accuracy of surface reconstruction. The number of triangles roughly corresponds to the amount of detail in the surface.

The volume of the gamut, usually expressed in cubic CIELAB units, can be used as a single figure-of-merit for comparison of different printers. In (Balasubramanian and Dalal, 1997) this quantity was computed by summing the volumes of tetrahedra formed by vertices of each surface triangle and a common point known to be inside the gamut and visible from all the triangles (CIELAB point $[50, 0, 0]$ often satisfies this condition). We propose to perform the calculation by summing the volumes of all tetrahedra which belong to the alpha-shape. This method does not depend on the existence of such a central point and can be applied to arbitrary gamut shapes including shapes consisting of sev-

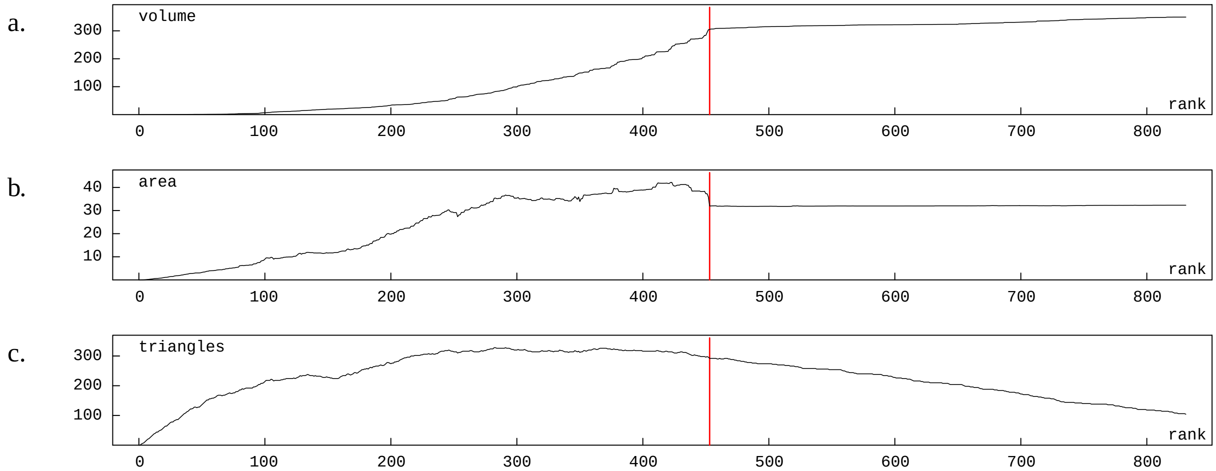

Figure 3: *Signatures of a laser printer gamut alpha-shape as a function of the alpha-rank: a. volume (in thousands of cubic* CIELAB *units), b. surface area (in thousands of square* CIELAB *units), and c. the number of triangles. The vertical line shows the alpha-rank corresponding to the value* α^* *for which the alpha-surface consists of a single component.*

eral disjoint components which are common in the case of image gamuts.

By an extension of this method, one can calculate the approximate percentage of colors shared by two different gamuts. The volumes of $V(A)$ and $V(B)$ of alpha-shapes based on the two separate data sets A and B and the volume of the combined data sets $V(A \cup B)$ are computed. Then the volume of the common part can be found from the equation $V(A \cap B) = V(A) + V(B) - V(A \cup B)$. One possible application of this method is to find the percentage of CRT colors that can be reproduced on a specific printer.

Figure 4 presents the results of applying our gamut surface construction method to a set of colors used in an image. The parameter α was chosen to be less than α^* in order to better emphasize lack of light blue colors in the image. In this case there is no central point from which all surface triangles are visible and therefore the gamut surface and its volume cannot be computed by the method described in (Balasubramanian and Dalal, 1997). This also makes it impossible to represent this gamut surface as a function of lightness and hue using method of Braun and Fairchild (1996).

Conclusions

The described method enables creation of approximate analytical descriptions of the surfaces of the gamuts of color printing devices and color images. This facilitates comparisons of gamuts, computation of simple figure-of-merit quantities related to the quality of the device such as a volume of a gamut, and performing out-of-gamut mappings using geometric techniques. Many printers exhibit natural color gamut boundaries which exceed the physical colorant boundaries and therefore it is important to use geometric methods of gamut surface construction to get an accurate estimate of the gamut boundary and volume.

As an extension of our method it is possible to construct a shape that has different levels of detail in different parts of space by assigning a weight to each point where a large weight favors and a small weight discourages connections to neighboring points. The resulting object is known as the weighted alpha shape (Edelsbrunner, 1992). If all weights are zero, it is the same as the original, unweighted alpha shape.

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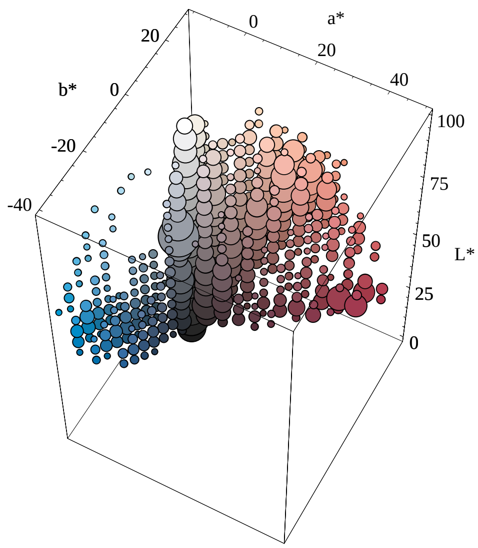
a.

b.



c.

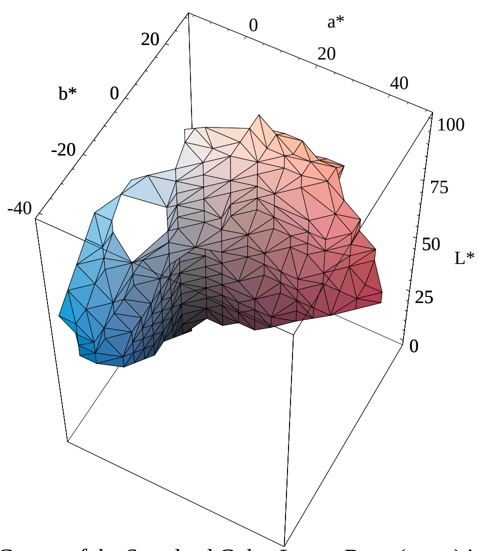


Figure 4: *Gamut of the Standard Color Image Data (*SCID*) image "N1A: Portrait": a. image; b. histogram of colors present in the image in* CIELAB *space (larger spheres correspond to colors appearing more frequently); c. alpha-shape of the set of points from subfigure b. for $\alpha < \alpha^*$.*

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