In *Proceedings of the 4th IEEE International Workshop on Cellular Neural Networks and Applications* (CNNA'96), pages 219–223, Seville, Spain, June 24–26, 1996

Chaotic CNN for Image Segmentation[1]

Andrzej Łozowski, Tomasz J. Cholewo*, Stanisław Jankowski†, and Mikołaj Tworek‡*

*Department of Electrical Engineering, University of Louisville
Louisville, Kentucky 40292, USA
fax: (502) 852-6807, e-mail: aglozo01@@starbase.spd.louisville.edu

†Institute of Electronic Fundamentals, Warsaw University of Technology
Nowowiejska 15/19, 00-665 Warsaw, Poland

‡Interdisciplinary Centre for Mathematical and Computational Modeling,
Warsaw University, Banacha 2, 02-097 Warsaw, Poland

ABSTRACT: *A chaotic* CNN *associative memory that is able to perform complex pattern separation is presented in this paper. The introduced model has the form of a network composed of chaotic oscillators locally coupled by nonlinear conductances. A local pseudoinverse learning rule for binary pattern storage in the cellular memory structure is proposed. The chaotic units can temporarily synchronize or anti-synchronize and hence retrieve patterns in the global synchronization state. Chaotic wandering from one pattern to another is an inherent property of the present model and allows object separation if a pattern superposition is presented to the network's input.*

1 Introduction

Synchronization phenomenon is a fundamental property of coupled chaotic oscillator arrays and has been broadly investigated recently. A portion of the inspiration for this research originates from biological systems [1]. Weakly interacting chaotic units in a homogeneous array can give rise to spontaneous pattern formation regarded as spatio-temporal chaos in a high order system. An example of such an effect is spiral wave formation in a CNN reaction diffusion model [2]. According to [3] sufficient interconnection strengths cause global array synchronization. This may be regarded as lowering the order of the whole system to the order of a single chaotic unit. In the perfectly synchronized state interactions between identical units disappear which allows the units to perform their unperturbed dynamic behavior. However, such a global synchronization is trivial from the point of view of information storage. Information can potentially be encoded in an array if at least two distinct synchronization modes are possible. As introduced in [4] synchronization and anti-synchronization states between symmetric attractors of Chua's circuits are made possible by employing nonlinear coupling conductances in place of constant-valued linear ones.

A network of chaotic oscillators can act as an associative memory if coupling interconnections between units are formed according to some learning technique. This leads to inhomogeneous coupling matrices such as the Hebbian matrix proposed in [4] for a fully connected Hopfield-like oscillatory structure. An interesting feature of such a memory model is chaotic wandering among patterns which are unstable equilibria of the network oscillatory state [5]. The network dynamics first approach these equilibria in the form of an imperfect global synchronization and then diverge from them due to the repelling forces between chaotic units. A stimulus corresponding to an input pattern produces a sequence of synchronization states corresponding to the stored patterns which are correlated with the input. The appearance frequencies of these synchronization states correspond to the magnitude of the correlation of the corresponding stored patterns with the input. Thus, considering the input as a superposition of the stored patterns, the network performs pattern separation.

In this paper a CNN of coupled Lorenz systems for pattern separation is introduced. Unlike the classic CNN [6] the presented model is a cellular associative memory [7] which stores patterns in connectivity matrices derived using a local pseudoinverse rule. The memory is shown to be able to retrieve binary patterns in the form of imperfect global synchronization states invoked by coupling interconnections. Numerical examples of pattern retrieval and pattern separation are provided.

[1]The video presentation to be shown at the conference has been prepared using the computer resources of the Interdisciplinary Centre for Mathematical and Computational Modeling, Warsaw University.

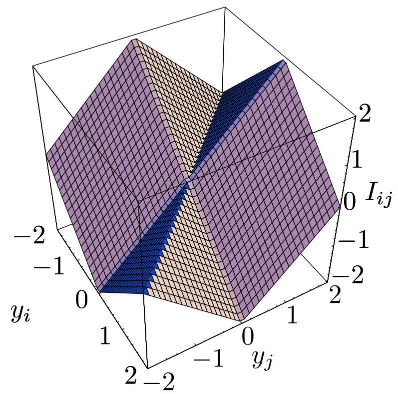

Figure 1: Nonlinear coupling function $I_{ij}(y_i, y_j, g_{ij})$ for $g_{ij} > 0$. Note that if $|y_i| = |y_j|$ the units are decoupled since $I_{ij} = 0$.

2 CNN of Lorenz systems

The proposed model of the cellular network is composed of N Lorenz systems. Each unit i is coupled with units from its neighborhood δ_i via the variable y_i. Node i equations have the following form:

$$\begin{array}{rcll} \dot{x}_i & = & \sigma(y_i - x_i) & \\ \dot{y}_i & = & -x_i z_i + r x_i - y_i & + \sum_{j\in\delta_i} I_{ij}(y_i, y_j, g_{ij}) + I_{i0} + n_i \\ \dot{z}_i & = & x_i y_i - b z_i. & \end{array} \tag{1}$$

Here, σ, r, and b are the Lorenz system parameters such that a single unit performs chaotic oscillation when not coupled.

If the coupling variables are considered as node voltages the network units interact with each other via currents I_{ij}. Current I_{ij} entering the ith node is understood to be the influence that node j has on node i. For linearly coupled units I_{ij} would simply equal $g_{ij}(y_j - y_i)$. A network with such couplings will globally synchronize after some transient time only if the coupling conductances g_{ij} have sufficient magnitude and the connectivity matrix satisfies conditions given in [3].

However, in order to obtain the possibility of two distinct synchronization modes, nonlinear coupling as proposed in [4] has been used in place of constant-valued conductances g_{ij}. The coupling current is expressed as

$$I_{ij}(y_i, y_j, g_{ij}) = g_{ij} y_j + |g_{ij}| y_i (|s_{ij}| - |s_{ij} + 1|) \tag{2}$$

where $s_{ij} = (y_j/y_i)\,\mathrm{sign}\, g_{ij}$. Note that I_{ij} is zero when $y_i = y_j$ or $y_i = -y_j$. Hence, units i and j become decoupled if their second state variables have identical or exactly opposite time behavior. Since the Lorenz attractor is symmetric the coupling expressed by Equation (2) allows node i to be synchronized or anti-synchronized with node j by suppressing I_{ij} in both cases. Note also that for positive g_{ij} (as shown in Figure 1) and for $y_i \approx y_j$, $I_{ij} = g_{ij}(y_j - y_i)$. In this case the coupling element acts as a linear conductance g_{ij} which forces unit i to synchronize in-phase with unit j. Conversely, for negative g_{ij}, $I_{ij} = g_{ij}(y_j + y_i)$ when $y_i \approx y_j$. Such a situation enhances anti-synchronization of the units. Both cases correspond to interactions found in the classic Hopfield model where positive connections make spins agree with each other whereas negative connections enforce opposite spins.

Each unit i additionally obtains an input stimulus $I_{i0}(y_i, y_0, g_{i0})$ from an external Lorenz oscillator. The input current I_{i0} is also expressed by Equation (2), where conductance $g_{i0} = k\xi_i g_0$. ξ_i is the ith entry in the input bipolar pattern ξ and the coefficient k adjusts input coupling strengths.

All Lorenz oscillators in the network have the same parameters σ, r, and b. Actual circuit implementation of the proposed model would obviously result in a spread of parameter values among units. This effect can be reasonably modeled by adding noise n_i to each network unit.

The typical CNN, as defined in [6], is an image processing device rather than an associative memory. In order to store patterns in a chaotic cellular network the local connectivity matrices need to be used in place of constant cloning templates. Following the Hebbian learning rule a set of p uncorrelated random bipolar patterns $\{\xi^\mu\}, 1 \leq \mu \leq p$ could be encoded in the network interconnections, assuming that all the weights not belonging to the local neighborhood of a neuron are ignored. However, the low efficiency of Hebb's rule combined with limited neighborhood sizes would result in an unacceptably low memory capacity. To provide reasonable memory capacity as well as correlated pattern storage, a local pseudoinverse learning is introduced.

The purpose of using the pseudoinverse rule [8] is to create the global connectivity matrix $\mathbf{G} = [g_{ij}]$ such that $\mathbf{G}$ is an identity transformation of patterns ξ^μ:

$$\forall \mu \quad \sum_{j \in \delta_i} g_{ij} \xi_j^\mu = \xi_i^\mu. \tag{3}$$

Most of the entries in $\mathbf{G}$ are equal to zero due to the cellular structure of the network. Only entries g_{ij} where $j \in \delta_i$ can have non-zero values. This can be obtained by applying the pseudoinverse rule locally for each node i. Let P_i be a subset of ξ^μ containing only those patterns that are linearly independent considering entries $j \in \delta_i$. Let $\mathbf{Q}^{\langle i \rangle}$ be a local pattern correlation matrix whose entries are $q_{\mu\nu}^{\langle i \rangle} = \sum_{j \in \delta_i} \xi_j^\mu \xi_j^\nu$ for $\mu, \nu \in P_i$. By the definition of P_i matrix $\mathbf{Q}^{\langle i \rangle}$ is invertible. Let $\mathbf{R}^{\langle i \rangle} = [r_{\mu\nu}^{\langle i \rangle}]$ be the inverse of $\mathbf{Q}^{\langle i \rangle}$. Entries g_{ij} satisfying Equation (3) can be found by using the following local pseudoinverse learning rule:

$$\forall j \in \delta_i \quad g_{ij} = g_0 \sum_{\mu \in P_i} \sum_{\nu \in P_i} \xi_i^\mu r_{\mu\nu}^{\langle i \rangle} \xi_j^\nu.$$

Note that patterns ξ^μ become eigenvectors of matrix $\mathbf{G}$ with the associated eigenvalues equal to 1. Besides these and some number of spurious patterns created by the pseudoinverse learning rule there exist eigenvectors corresponding to zero eigenvalues or generally complex ones small in magnitude. Taking into account only the meaningful eigenvalues the synchronization properties of the network can be shown by the following theorems introduced in [3]. Neglecting the input signals and noise terms in Equation (1) and assuming that the network is in a synchronization state $\dot{y}_i = f_{y_i}(x_i, y_i, z_i) - y_i \sum_{j \in \delta_i} g_{ij} + \sum_{j \in \delta_i} g_{ij} y_j$. Define the system Lyapunov function as:

$$V(\mathbf{y}(t)) \triangleq -\frac{1}{2} \mathbf{y}^T(t)\, \mathbf{G}\, \mathbf{y}(t). \tag{4}$$

If the system is synchronized $y_i(t)$ can be represented as a product of two functions $y_i(t) = \omega(t) s_i(t)$ where $\omega(t)$ is the time behavior of the autonomous variable y of the Lorenz system and $s_i(t)$ is temporarily 1 or -1, depending on the ith entry in the pattern being retrieved by the network. Thus the Lyapunov function in Equation (4) can be approximated as $V(\mathbf{y}) \approx -\frac{1}{2}\, \omega^2(t)\, \mathbf{s}^T(t)\, \mathbf{G}\, \mathbf{s}(t)$. If the pattern $\mathbf{s}$ is an eigenvector of $\mathbf{G}$ the approximation $V(\mathbf{y}) \approx -\frac{1}{2} \mathbf{y}^T(t)\, \mathbf{y}(t)$ is valid. Then the time derivative of $V(\mathbf{y})$ has the simple form of:

$$\dot{V}(\mathbf{y}(t)) = -\mathbf{y}^T(t)\, \dot{\mathbf{y}}(t).$$

Linearization of Equation (1) about the synchronization state involves $N \times 3$ linear equations where the system Jacobian including the terms $\mathbf{A}\, \mathbf{y}$ and $\mathbf{G}\, \mathbf{y}$ is the derivative linear matrix operator. Thus the Lyapunov function time derivative simplifies to the following form:

$$\dot{V}(\mathbf{y}(t)) = -\mathbf{y}^T(t)[\mathbf{A}\, \mathbf{y}(t) + \mathbf{G}\, \mathbf{y}(t)] = -\mathbf{y}^T(t)\, \mathbf{A}\, \mathbf{y}(t) - \mathbf{y}^T(t)\, \mathbf{G}\, \mathbf{y}(t).$$

According to the Lyapunov direct method [3] the system is asymptotically synchronized if the time derivative $\dot{V}$ is negative. With a positive definite matrix $\mathbf{G}$ this condition is satisfied as long as the network coupling strength parameter g_0 remains sufficiently large.

Note that the eigenvectors of $\mathbf{G}$ associated with small magnitude eigenvalues have been omitted in the Lyapunov function approximation. Thus, there may exist unstable directions in the system phase space causing the system to leave the synchronization states. In the next section numerical simulations demonstrate that the network synchronizes temporarily in a certain pattern, desynchronizes after some time, and then moves to another synchronization state. Synchronization states are usually not ideal but allow pattern identification by comparing the behavior of the network units.

3 Computational Experiments

As opposed to non-oscillatory associative memories, temporal response to a presented input pattern is an inherent property of the introduced memory model. The dynamical performance depends on a set

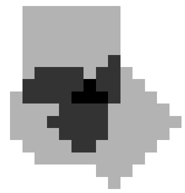

Figure 2: Three overlapping patterns ($\square$, $\bigcirc$, and $\diamond$) stored in the associative memory.

of parameters such as coupling conductance magnitude g_0, input strength coefficient k, and the level of noise n_i. To illustrate the capabilities of the model a network composed of $N = 15 \times 15$ units has been simulated. Each of the units has been coupled with its eight closest neighbors using array periodic boundary conditions. Three bipolar patterns, referred to as $\square$, $\bigcirc$, and $\diamond$ have been stored in coupling interconnections by means of the local pseudoinverse learning rule. The patterns are shown in Figure 2. Note that the patterns are overlapping as indicated by various grey-levels. The patterns possess the following mutual correlations:

	$\square$	$\bigcirc$	$\diamond$
$\square$	1	0.271	0.0667
$\bigcirc$	0.271	1	0.333
$\diamond$	0.0667	0.333	1

First, the network was presented with a corrupted pattern $\diamond$ shown in Figure 3. This input was correlated with each of the stored patterns as follows:

	$\square$	$\bigcirc$	$\diamond$
input	0.138	0.262	0.911

A sequence of temporary synchronization states is the network response to this stimulus. The network was considered to be globally synchronized when the average distance between coupling variable absolute values was smaller than an arbitrary threshold η which can be expressed as an inequality $\langle |\langle |y_i|\rangle - |y_i|| \rangle < \eta$. Each synchronization state, regarded as the network response, corresponded to some output pattern. This wandering from pattern to pattern in an unpredictable sequence allowed the network to explore all possible states resembling the input pattern. In a sequence of 100 successive synchronizations pattern $\diamond$ in its undistorted form was found most frequently (43 times), whereas the remaining two stored patterns never appeared in this sequence. This behavior demonstrates the ability of the memory to restore patterns.

Different behavior occurs when the input pattern is correlated with two of the patterns stored in the memory. The network was presented with a superposition of patterns $\square$ and $\bigcirc$ as shown in Figure 4. Correlations between this input and the stored patterns are as follows:

	$\square$	$\bigcirc$	$\diamond$
input	0.689	0.582	0.0222

Again the network responded with a sequence of temporary synchronizations. In a sequence of 100 successive synchronization states the network retrieves patterns $\square$ and $\bigcirc$ (4 and 5 times respectively) and avoids pattern $\diamond$. Thus the superposition of two patterns was decomposed into two distinct components.

4 Conclusions

An associative memory for pattern separation task by means of chaotic wandering among patterns has been presented in this paper. The electronic implementation of the proposed network seems feasible due to its cellular structure. In place of the previously utilized Lorenz system, generalized Chua's circuits can be employed without significant changes to the memory performance [9]. The only requirement for the units is symmetry of their dynamic attractors.

The presence of both attracting and repelling forces in the neighborhoods of accessible synchronization states enables pattern wandering without the need for global "reset" units or additional noise. However,

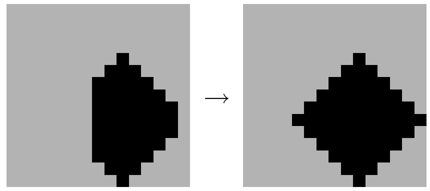

Figure 3: Pattern completion ability. Corrupted input pattern causes the network to frequently synchronize in a state corresponding to the undistorted stored pattern $\diamond$.

Figure 4: Pattern decorrelation. When presented with a superposition of patterns $\Box$ and $\bigcirc$ the network responds with a sequence of synchronization states in which the original patterns $\Box$ and $\bigcirc$ are found.

this effect can be globally controlled by adjusting input couplings strengths. Correlation between an input and a memorized pattern results in dynamic recall of this pattern within the network dynamics.

The proposed memory model should be considered as a network with strong interactions. Such a system has an order close to the low order of a single oscillator when in a synchronization state. Thus pattern formation is fully determined by the global connectivity matrix and the input pattern regardless of initial conditions used for simulations. Hence this memory model is strictly related to the classic neural associative memory but additionally exhibits complex dynamic behavior.

References

[1] C. A. Skarda and W. J. Freeman, "How brains make chaos in order to make sense of the world," *Behav. Brain. Sc.*, vol. 10, pp. 161–195, 1987.

[2] R. R. Klevecz, J. Bolen, and O. Duran, "Self-organization in biological tissues: analysis of asynchronous and synchronous periodicity, turbulence, and synchronous chaos emergent in coupled chaotic arrays," *International Journal of Bifurcation and Chaos in Applied Sciences and Engineering*, vol. 2, pp. 941–953, December 1992.

[3] C. W. Wu and L. O. Chua, "Synchronization in an array of linearly coupled dynamical systems," *IEEE Transactions on Circuits and Systems I: Fundamental Theory and Applications*, vol. 42, pp. 430–447, August 1995.

[4] S. Jankowski, A. Londei, C. Mazur, and A. Lozowski, "Synchronization and association in a large network of coupled Chua's circuits," *Int. J. Electronics*, vol. 79, no. 6, pp. 823–828, 1995.

[5] P. Thiran and M. Hasler, "Information processing using stable and unstable oscillations: a tutorial," in *CNNA-94 Third IEEE International Workshop on Cellular Neural Networks and their Applications*, (Rome, Italy), pp. 127–136, December 18–21, 1994.

[6] L. O. Chua and L. Young, "Cellular neural networks: Theory," *IEEE Transactions on Circuits and Systems*, no. 35, pp. 1257–1272, 1988.

[7] M. Balsi, "Generalized CNN: potentials of a CNN with non-uniform weights," in *2nd Workshop on CNN and their Applications*, (Munich), pp. 129–134, 1992.

[8] J. Hertz, A. Krogh, and R. G. Palmer, *Introduction to the Theory of Neural Computation*. Boston: Addison–Wesley, 1991.

[9] R. Brown, "Generalization of the Chua equations," in *IEEE CAS I*, no. 40, pp. 878–883, 1993.