International Journal of Control and Automation
Vol.7, No.2 (2014), pp.65-74
http://dx.doi.org/10.14257/ijca.2014.7.2.07

A Nonlinear Regression based Approach for Multilayer Soil Parameter Estimation

Min-Jae Kang[1], Chang-Jin Boo[1], Ho-Chan Kim[1] and Jacek M. Zurada[2]

[1] Department of Electrical Engineering, Jeju National University, Korea
[2] Department of Electrical Engineering, University of Louisville, USA

[1]{minjk, boo1004, hckim}@jejunu.ac.kr, [2]j.zurada@ieee.org

Abstract

The estimation of soil parameters of multilayer structure leads to useful information for designing a safe grounding system. This paper presents a nonlinear regression based estimation scheme to extract soil parameters from the kernel function of apparent earth resistivity. The kernel function of apparent earth resistivity can be obtained from the measured apparent earth resistivity data. The performance of the proposed method has been verified by carrying out a numerical example.

Keywords: *Estimation, soil parameters, grounding system, nonlinear regression, soil resistivity, kernel function, apparent earth resistivity*

1. Introduction

It is important to know the earth structure in the given area when the grounding system is designed. Because badly designed grounding system cannot ensure the safety of equipment and personnel [1]. The soil is modeled as a uniform medium in early researches and the simplified formula is used to estimate the resistance of grounding system. For simplifying the problem, in a host of engineering application, multilayer soils are modeled by N horizontal layers with distinct resistivity and depths [2]. A Wenner configuration method is well known to measure the soil resistivity for this simplified earth model.

The inversion of soil parameter is an unconstrained nonlinear minimization problem [3, 4]. Supposing there are N different layers below the ground surface then 2N-1 parameters need to be determined, because there exists different N-1 thicknesses and N resistivity in the Wenner configuration model. Therefore, many different optimization methods have been carried out to invert soil parameters in the hope of improving the performance. However, there exist two difficulties in inverting the soil parameters using optimization methods. On one hand, it is hard to obtain the derivatives of the optimized expression. On the other hand, the computing time is hugely consumed. These difficulties can be solved efficiently by using the proposed method in this paper.

This paper presents a regression scheme to estimate soil parameters from the kernel function of the apparent earth resistivity. This regression based estimation scheme is composed of two steps: first, it is needed to obtain the kernel function of apparent earth resistivity from the measured apparent resistivity data, and secondly from this kernel function, the regression method is applied to estimate soil parameters. The obtaining kernel function method has been showed in J. Zou *et al.* [2]. In this paper J. Zou‘s method is modified more efficiently using linearization method for obtaining the kernel.

A three-layer earth structure has been used in a numerical example for simplicity to examine the proposed method, however a multi-layer structure more than three can be applied

ISSN: 2005-4297 IJCA

to use this method. The rest of this paper is organized as follows: Section 2 gives a brief overview of the apparent soil resistivity for a multi-layer earth model using Wenner method. Section 3 presents an analytic formulation to invert soil parameters using the kernel function and a numerical example result, and the conclusions are presented in Section 4.

2. Wenner 4-point Test and Apparent Resistivity

A schematic diagram of the apparent resistivity measurement setup has been shown in Figure 1. $h_i\ (i=1,2,\cdots,N-1)$ and $\rho_i\ (i=1,2,\cdots,N)$ are the thickness and the resistivity of the th layer respectively for an N–layer soil structure. A current I is injected into the soil by applying the power between electrodes A and B and the potential difference V between electrodes C and D is measured. By changing the test

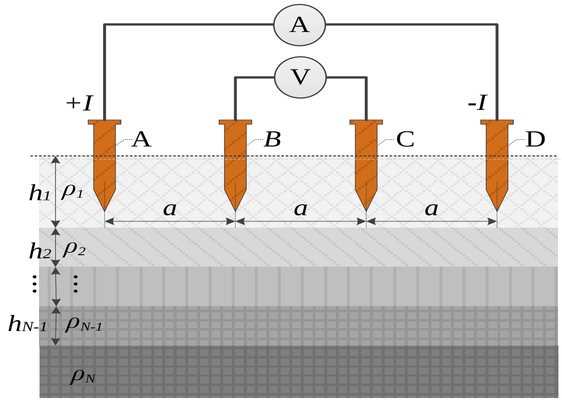

Figure 1. Wenner configuration for measuring apparent soil resistivity of N-layer earth structure

electrode span a, a set of apparent resistivity curves varying with electrode span can be obtained. If a point current source enters at a point A on the surface, the Laplace differential equation can be used to describe the potential distribution by using cylindrical coordinates, r, θ, z. In this case, Laplace differential equation becomes that there is cylindrical symmetry and so θ is eliminated,

$$\frac{\partial V^2}{\partial r^2}+\frac{1}{r}\frac{\partial V}{\partial r}+\frac{\partial V^2}{\partial z^2}=0 \quad \textbf{(1)}$$

With appropriate boundary conditions, the surface potential at z=0 becomes

$$V(r)=\frac{I\rho_1}{2\pi}\left[\int_0^{\infty}\left(1+2f(\lambda)\right)J_0(\lambda r)d\lambda\right] \quad \textbf{(2)}$$

Where ρ_1 is the soil resistivity of the first layer, $J_0(\lambda r)$ is the zero order Bessel's function of the first kind and the kernel function $f(\lambda)$ is as follows [1]

$$f(\lambda)=\alpha_1(\lambda)-1 \quad \textbf{(3)}$$

$$\alpha_1(\lambda) = 1 + \frac{2K_1 e^{-2\lambda h_1}}{1 - K_1 e^{-2\lambda h_1}} \qquad K_1(\lambda) = \frac{\rho_2\alpha_2 - \rho_1}{\rho_2\alpha_2 + \rho_1}$$
$$\alpha_2(\lambda) = 1 + \frac{2K_2 e^{-2\lambda h_2}}{1 - K_2 e^{-2\lambda h_2}} \qquad K_2(\lambda) = \frac{\rho_3\alpha_3 - \rho_2}{\rho_3\alpha_3 + \rho_2}$$
$$\vdots \qquad\qquad \vdots$$
$$\alpha_{n-1}(\lambda) = 1 + \frac{2K_{n-1} e^{-2\lambda h_{n-1}}}{1 - K_{n-1} e^{-2\lambda h_{n-1}}} \qquad K_{n-1}(\lambda) = \frac{\rho_n - \rho_{n-1}}{\rho_n + \rho_{n-1}}. \tag{4}$$

If the potential difference ΔV_{CD} between potential electrodes C and D in Figure 1 is tested, the apparent resistivity $\rho(a)$ can be expressed by

$$\rho(a) = 2\pi a \frac{\Delta V_{CD}}{I} = 2\pi a \frac{[V(a) - V(2a)]}{I} \tag{5}$$

Where $V(a)$ and $V(2a)$ are the potential of electrodes C and D generated by current test electrodes A and B respectively. By substituting (2) into (5), one can obtain

$$\rho(a) = \rho_1 \left[1 + 2a\int_0^\infty f(\lambda)[J_0(\lambda a) - J_0(2\lambda a)]d\lambda \right] \tag{6}$$

3. Estimation of Soil Parameters

As seen in (4), with the assumed N-layer earth soil parameters, $K_{n-1}(\lambda)$ is calculated first, then $\alpha_{n-1}(\lambda)$, $K_{n-2}(\lambda)$, …. $K_1(\lambda)$, $\alpha_1(\lambda)$ are calculated in reverse order. With the last calculated $\alpha_1(\lambda)$, the kernel function $f(\lambda)$ can be formulated as in (3). This kernel function $f(\lambda)$ is used in (2) to evaluate the apparent resistivity $\rho(a)$. The assumed N-layer earth soil parameters are changed continuously and the apparent resistivity $\rho(a)$ is calculated until this evaluated $\rho(a)$ is as close as possible to the measured apparent resistivity by Wenner method.

This section will describe an analytical formulation of the inverse solution to the soil parameter estimating problem using Wenner method. The analytical formulation is developed by analyzing the kernel function of the integral equation of apparent resistivity. First, the kernel function is obtained from the data of measured apparent resistivity using Wenner method. Second, using the kernel function obtained in the first step, the proposed formulation in this paper provides the estimation of thickness and resistivity of the earth layers. The structure of earth layers can then be determined from the respective soil parameter estimates.

3.1. Inversion of the kernel function using the measured apparent resistivity

J. Zou *et al.* [2] showed that the kernel function of apparent earth resistivity can be obtained from the data measured by Wenner configuration method. In his method, the kernel function $f(\lambda)$ is assumed to be a continuous and smooth function with respect to λ, and the property of its asymptotic function is much similar to the damping exponential function [2, 5]. So the kernel function $f(\lambda)$ can be expressed by a series of exponential function as

$$f(\lambda) \approx \sum_{k=1}^{k=\infty} b_k e^{-c_k \lambda} \quad (7)$$

where b_k, c_k are constant.
Using the following Lipschitz integrating

$$\int_0^{\infty} e^{-\lambda|c|} J_0(\lambda l) d\lambda = \frac{1}{\sqrt{c^2 + l^2}} \quad (8)$$

the apparent resistivity $\rho(a)$ in (6) can be approximated as follows

$$\rho(a) \approx \rho_1 \left\{ 1 + 2a \sum_{k=1}^{N} b_k \left[\frac{1}{\sqrt{c_k^2 + a^2}} - \frac{1}{\sqrt{c_k^2 + 4a^2}} \right] \right\} \quad (9)$$

To find b_k and c_k from the measured apparent resistivity data, rearranging (9) and replacing $\rho(a)$ with the measured $\rho^m(a)$ gives

$$\sum_{k=1}^{N} b_k \left[\frac{1}{\sqrt{{c_k}^2 + {a_i}^2}} - \frac{1}{\sqrt{{c_k}^2 + 4{a_i}^2}} \right] = \frac{1}{2a_i} \left(\frac{\rho^m(a_i)}{\rho_1} - 1 \right), (i = 1, 2, ..., M) \quad (10)$$

Where $\rho(a_i)$ ($i = 1 \cdots M$) is the measured apparent resistivity with the test electrode span a_i ($i = 1 \cdots M$) of the Wenner configuration method and c_k is the k th decay constant corresponding to the k th sampling of integral [2].

3.2. Determining Kernel Function by Linearization method

In this section the new algorithm has been suggested how to linearize J. Zou's method for obtaining the kernel function efficiently. Equation (10) is a nonlinear system and this kind of nonlinear equation can be solved by versatile iterative methods including Newton-Raphson method [7, 8]. However, this case is difficult to use general iterative methods because a large number of b_k and c_k are required for finding the correct kernel function $f(\lambda)$ in (7). It has been known that it is difficult to find a precise solution of a large number variables involving nonlinear system.

Assuming $f(\lambda)$ in (7) can be transformed as follows

$$f(\lambda) \approx \sum_{k=1}^{N} b_k e^{-d \times k \times \lambda} \quad (11)$$

Then (11) becomes a linear system because left hand side bracket in (10) is determined. Where d is very small constant, 0.1 is good enough for d according to the author's experience. Then (10) can be expressed as a linear system as follows

$$\begin{bmatrix} A_{11} & A_{12} & \cdots & A_{1N} \\ A_{21} & A_{22} & \cdots & A_{2N} \\ \vdots & \vdots & \cdots & \vdots \\ A_{M1} & A_{M2} & \cdots & A_{MN} \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_N \end{bmatrix} = \begin{bmatrix} \dfrac{1}{2a_1}\left(\dfrac{\rho^m(a_1)}{\rho_1}-1\right) \\ \dfrac{1}{2a_2}\left(\dfrac{\rho^m(a_2)}{\rho_1}-1\right) \\ \vdots \\ \dfrac{1}{2a_M}\left(\dfrac{\rho^m(a_M)}{\rho_1}-1\right) \end{bmatrix} \quad (12)$$

where

$$A_{ik} = \frac{1}{\sqrt{d^2k^2 + a_i^2}} - \frac{1}{\sqrt{d^2k^2 + 4a_i^2}} \quad (13)$$

Because a_i is constant which is the i-th test electrode span, A_{ik} is also constant. Nevertheless, the number of equations(N) is not same as the number of variables(M), in other words ($M \neq N$), therefore the unique solution is not possible. After trials and errors, it is found that the kernel function $f(\lambda)$ can be obtained correctly only in the case of N □ M. This system is an underdetermined system where many possible solutions exist. QR factorization is one of many methods to solve an underdetermined system. QR factorization is used to solve in this paper.

3.3. Inversion Soil Parameters using a Nonlinear Regression Method

The proposed formulation provides the analytic method for estimating thickness and resistivity of the earth layers. The characteristic of the kernel function is analyzed to develop the analytic method for estimating soil parameters.

As seen in (4), $K_i(\lambda)$ is a function of ρ_i, ρ_{i+1} and $\alpha_{i+1}(\lambda)$. And because $\alpha_{i+1}(\lambda)$ converges to 1 as λ increases, $K_i(\lambda)$ converges as follows

$$\overline{K_i}(\lambda) = \lim_{\lambda \to increase} K_i(\lambda) = \frac{\rho_{i+1} - \rho_i}{\rho_{i+1} + \rho_i} \quad (14)$$

By rearranging the equation of left hand side in (4), also $K_i(\lambda)$ can be derived as follows

$$K_i(\lambda) = \Omega_i(\lambda) e^{2\lambda h_i} \quad (15)$$

where

$$\Omega_i(\lambda) = \frac{\alpha_i(\lambda) - 1}{\alpha_i(\lambda) + 1}$$

To make $K_i(\lambda)$ in (15) converge to same value as (14), $\Omega_i(\lambda)$ has to be the exponentially decreasing form canceling out the exponential part in (15) as follows

$$\overline{\Omega_i}(\lambda) = \lim_{\lambda \to increase} \Omega_i(\lambda) = c_i e^{-2\lambda h_i} \quad \textbf{(16)}$$

Then c_i becomes as

$$c_i = \frac{\rho_{i+1} - \rho_i}{\rho_{i+1} + \rho_i}$$

Therefore, the i th layer thickness h_i and c_i can be estimated from $\Omega_i(\lambda)$ using nonlinear regression method. Eq. (16) can be linearized by taking its natural logarithm to yield

$$\ln \overline{\Omega_i}(\lambda) = \ln c_i - 2h_i\lambda \quad \textbf{(17)}$$

Thus, the plot of $\ln \overline{\Omega}_i(\lambda)$ versus λ will yield a straight line with a slope of $-2h_i$ and an intercept of $\ln c_i$. Therefore, the i th layer thickness h_i and c_i can be estimated from the slop and intercept of the plot of $\ln \overline{\Omega}_i(\lambda)$.

As seen in (3), α_1 is obtained from the determined kernel function $f(\lambda)$, therefore c_1 and h_1 can be obtained which are the ratio of the first and second layer's resistivity and the first layer's thickness respectively. At the same time when h_1 is determined, also the correct $K_1(\lambda)$ is determined from (15). And then α_2 can be obtained from the right hand side equation of (4) as follows

$$\alpha_{i+1}(\lambda) = -\frac{\rho_i}{\rho_{i+1}} \frac{K_i(\lambda)+1}{K_i(\lambda)-1} \quad \textbf{(18)}$$

In this manner, all $K_i(\lambda)$ and $\alpha_i(\lambda)$ can be obtained iteratively and so are all the soil parameters ($h_1, h_2, \ldots, h_{n-1}$, ρ_1 $\rho_1, \rho_2, \ldots, \rho_n$).

3.4. Calculation Results

For the numerical example, a three-layer earth model with the soil parameters (h_1, h_2, ρ_1, ρ_2 and ρ_3) was considered as shown in Table 1.

Table 1. Parameters of a three-layer earth structure

Layer No	Resistivity(Ω .m)	Thickness(m)
1	165	2.2
2	1510	13.5
3	260	∞

Figure 2 shows that the kernel functions $f(\lambda)$ of a three layer earth model where the solid line is the exact kernel function determined from (3) and (4), and the dotted line is obtained by (11) which is a linearization method. The exact kernel function can be obtained with the known soil parameters in Table 1. As shown in Figure 2, the obtained kernel function by the proposed method is very close almost same to the exact one.

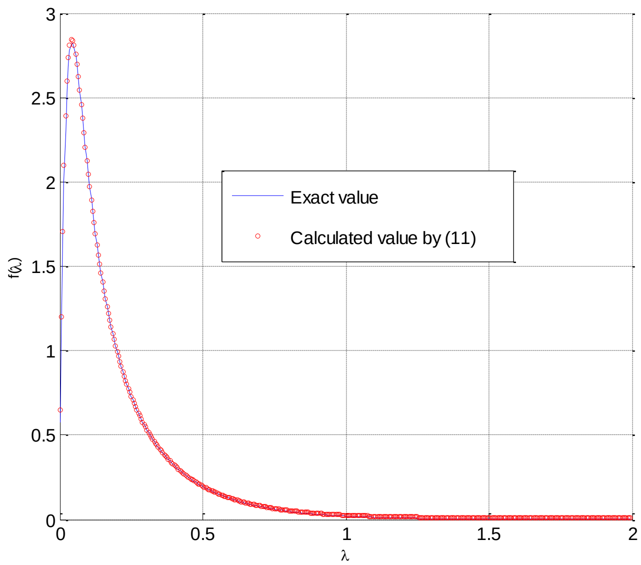

Figure 2. The kernel functions of 3-layer earth structure

Next step is to obtain the soil parameters from the kernel function $f(\lambda)$. To estimate the first layer depth h_1, and the ratio of first and second layer's resistivity c_1, the natural logarithm of Ω_1 is needed as follow

$$\ln(\Omega_1) = \ln\left(\frac{\alpha_1(\lambda)-1}{\alpha_1(\lambda)+1}\right) = \ln\left(\frac{f(\lambda)}{f(\lambda)+2}\right) \quad \textbf{(19)}$$

The solid line in Figure 3 is the natural logarithm of Ω_1 and the dotted line is the best fit-line. As shown in Figure 3, the best fit-line yield a straight line with a slope of -4.43 ,which is $-2h_1$, and an intercept of -0.22, which is $\ln c_1$. From these values, h_1 and c_1 are estimated as 2.215 and 0.8 respectively. Those values are really close to the exact ones of h_1 and c_1, which are 2.2 and 0.802 respectively. With this same manner, $K_2(\lambda)$, $\alpha_2(\lambda)$ and $\Omega_2(\lambda)$ can be obtained iteratively and so are all the soil parameters ($h_1, h_2, \rho_1, \rho_2, \rho_3$).

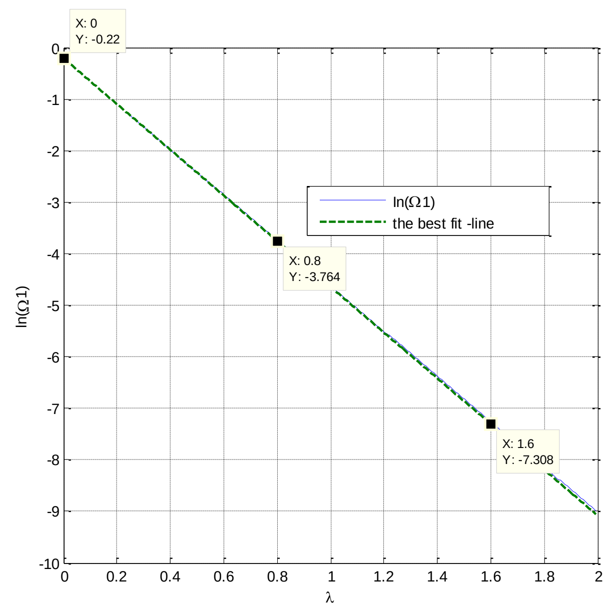

Figure 3. The natural logarithm of Ω_1 and the best fit-line

4. Conclusion

The determination of the soil parameters from Wenner's test data is an inverse problem. For the three-layer model, it is a five-variable optimization problem. In this paper, the analytic method rather than a general optimization algorithm is presented to invert the soil parameters.

This proposed method is formulated based on the characteristic of the kernel function of apparent resistivity. Also, this method is composed of two steps which are obtaining the kernel function and then estimating the soil parameters from the kernel function. In this paper, the first step has been improved by linearizing the nonlinear system and in the second step, a nonlinear regression method has been suggested for estimating soil parameters from the kernel function.

The robustness of the proposed algorithm has been verified by carrying out numerical simulation for three-layer model. The results show a promising performance of the method.

Acknowledgements

This research was supported by Basic Science Research Program through the National Research Foundation of Korea (NRF) funded by the Ministry of Education, Science and Technology (2010-0025438).

References

[1] H. R. Seedher and J. K. Arora, "Estimation of two-layer soil parameters using finite Wenner resistivity expression", IEEE Trans. on Power Delivery, vol. 7, (**1992**), pp. 1213-1217.
[2] J. Zou, J. L. He, R. Zeng, W. M. Sun and S. M. Chen, "Two-Stage Algorithm for Inverting structure Parameters of the Horizontal Multilayer Soil, IEEE Trans. on Magnetics (doi:10.1109/TMAG.2004.825013), vol. 40, (**2004**), pp. 1136—1139.

[3] J. L. Del Alamo, “A comparison among eight different techniques in two-layered earth”, IEEE Trans. on Power Delivery, vol. 8, (**1993)**, pp. 1890-1899, doi:10.1109/61.248299.

[4] F. P. Dawalibi, “Electromagnetic Fields generated by overhead and buried short conductors, IEEE Trans. on Power Delivery, vol. 1, (**1986**), pp. 105-119, doi:10.1109/TPWRD.1986.4308037.

[5] B. Zhang, X. Cui, L. Li and J. L. He, “Parameter Estimation of Horizontal Multilayer Earth by Complex Image Method”, IEEE Trans. on Power Delivery, vol. 20, (**2005**), pp. 1394-1401, doi:10.1109/TPWRD.2004.834673.

[6] F. P. Dawalibi, “Electromagnetic Fields generated by overhead and buried short conductors”, IEEE Trans. on Power Delivery, vol. 1, **(1986)**, pp. 105-119.

[7] S. C. Chapra, “Applied Numerical Methods with MATLAB for Engineering and Scientists”, 2nd ed, McGrow-Hill, NewYork, (**2008**).

[8] H. S. Yazdi, M. Arghiani and E. Nemati, “Nonlinear Regression Model of a Human H and Volume: A Nondestructive Method”, International Journal of Control and Automation, vol. 4, no. 2, (**2011**) June, pp. 111-124.

Authors

Min-Jae Kang

He received his B.S. degree in Electrical Engineering from Seoul National University, Korea, in 1982, M.S. and Ph.D. degrees in Electrical Engineering from University of Louisville in 1989 and 1991, respectively. Since 1992, he has been with the Department of Electronic Engineering at Jeju National University, where he is currently a professor. He was a Visiting Scholar at the University of Illinois at Urbana-Champaign in 2003. His research interests include neural networks, grounding systems, and wind power control.

Chang-Jin Boo

He received his B.S., M.S., and Ph.D. degrees in Electrical Engineering from Jeju National University in 2001, 2003 and 2007, respectively. Since 2011 he has been working at the Research Institute of Advanced Technology at Jeju National University. His research interests include grounding system design and power system control.

Ho-Chan Kim

He received his B.S., M.S., and Ph.D. degrees in Control and Instrumentation Engineering from Seoul National University in 1987, 1989, and 1994, respectively. He was a research staff member from 1994 to 1995 at the Korea Institute of Science and Technology (KIST). Since 1995, he has been with the Department of Electrical Engineering at Jeju National University, where he is currently a professor. He was a Visiting Scholar at the Pennsylvania State University in 1999 and 2008. His research interests include smart grid, electricity market analysis, and control theory.

Jacek M. Zurada

He received his MS and PhD degrees (with distinction) in electrical engineering from the Technical University of Gdansk, Poland. Since 1989 he has been a Professor with the Electrical and Computer Engineering Department at the University of Louisville, Kentucky. He was Department Chair from 2004 to 2006. His interests include extension of complex-valued neurons to associative memories and percepton networks, sensitivity concepts applied to multilayer neural networks, application of networks for clustering. Dr. Zurada has served the profession and the IEEE in various elected capacities, including as President of IEEE Computational Intelligence Society in 2004-05. He has been member and chair of various IEEE CIS and IEEE TAB committees, including the IEEE TAB Periodicals Committee and IEEE TAB Periodicals Review Committee. He was the Founding Chair of the NNC Neural Networks Technical Committee. In 2012-13 he serves as IEEE TAB Periodicals Review and Advisory Committee Chair and in 2013 as VP-Technical Activities Elect