Cross-layer Design of Joint Beamforming and Random Network Coding in Wireless Multicast Networks

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***Abstract*—In this letter we consider a wireless network where the transmitter, such as a base station, multicasts hard-deadline-constrained prioritized data to users over fading channels. We propose a new cross-layer transmission scheme that involves simultaneous smart antenna beamforming at physical layer and random network coding at network layer to maximize the overall network throughput. In this joint design, beamforming is used for dynamic transmitting power allocation to achieve power gain; while the random network coding is adopted to achieve coding gain. The simulation results show the effectiveness of the proposed algorithm.**

***Index Terms*—cross-layer design, hard deadline constraint, prioritized data, beamforming, multicast, random network coding.**

I. Introduction

As wireless networks such as mobile internet quickly develop, higher level quality of service (QoS) in multimedia applications is gaining more popularity. To meet users' real-time QoS requirements, scalable compression techniques such as Scalable Video Coding (SVC) [1] are used to partition a video bit stream into a base layer and several enhancement layers. The base layer is the most important layer and must be present in order to have a reasonable video quality. The enhancement layers are organized in a hierarchical fashion such that the first enhancement layer must be present for the second enhancement layer to be useful. Then the second enhancement layer must be present for the third enhancement layer to be useful, and so on. Naturally, this wireless transmission scenario leads to the general multicast of Hard Deadline Constrained Prioritized (HDCP) Data [4].

Notably, network coding (NC) [2] offers a promising platform for multicast transmissions. It has been proved that one can approach the multicast capacity by using Random Network Coding (RNC) technique [3], by which a user can decode the original data when it receives a full set of independent linear coded packets. However, when the quality of the wireless channel is in deep fading, RNC may lead to poor performance when a full set of the coded packets can not be successfully received. The situation may get worse in HDCP data transmission due to the limited delivery time and data interdependencies.

To the best of our knowledge, the existing works on multicasting HDCP data with NC usually address the problem at the network layer only. In fact, when the transmitter multicasts the coded packet by an omnidirectional antenna, the transmitting power is wasted at the regions where no users exist. Moreover, if a coded packet from the transmitter cannot bring any benefit to some user(s) (e.g. the packet is redundant for decoding), there is no need to allocate transmitting power to these users at the current time slot. Additionally, different users are often at various distances from the transmitter, which causes big performance variations among users. As a result, the user(s) with higher Packet Error Rate (PER) become a "*bottleneck*" and reduce the overall network performance significantly. Therefore, it is necessary to explore the physical layer advantages and consider dynamic power allocation to different users at each time slot. Toward this end, we incorporate the physical layer beamforming (BF) into multicast scheduling. In the former works of BF in multicast, [13] applied beamforming into multicast but it did not consider RNC. [14] combined BF with application-layer coding to maxmized sum rate. Specifically in our letter, we jointly use BF for dynamic allocation of beam power and RNC at the network layer to make scheduling decisions based on the framework of Markov Decision Processes (MDP). Under this new cross-layer design, we not only achieve coding gain but also power gain to maximize the overall network throughput for HDCP Data.

II. System Model

For the convenience of illustration, we consider a wireless network consisting of one base station (BS) and multiple users, where BS is equipped with smart antennas to simultaneously generate multiple transmitting beams. As shown in Fig. 1, each beam has different transmitting power and points to different user (or user group)[1]. The BS transmits the network coded packets to all target users within a deadline of T_s time slots (each time slot corresponds to one coded packet transmission).

Here we use a special RNC to encode the HDCP data before sending them out. Specifically, when the length of the HDCP data is L, the BS creates L generations and each generation G_l $(1 \leq l \leq L)$ consists of l consecutive original HDCP data $\{a_1, a_2, \ldots, a_l\}$(the priority order is $a_1 \geq a_2 \geq \cdots \geq a_l$). The generation G_l can output the network coded packet c_l, which is the linear combination of the HDCP data in G_l. For example, when $L = 3$, the network coded packets from the three generations are $G_1 : c_1 = \beta_{11}a_1; G_2 : c_2 = \beta_{12}a_1 + \beta_{22}a_2; G_3 : c_3 = \beta_{13}a_1 + \beta_{23}a_2 + \beta_{33}a_3$. β_{ij} is randomly withdrawn from the finite field $\mathbb{F}_q$.

[1]The number of beams formed by smart antenna may be limited. For large number of users, each beam can point to a group of users who share similar network or channel conditions.

This work is partially supported by XXX.
Author 1 is XXX. Author 2 is XXX. Author 3 is XXX. Author 4 is XXX.

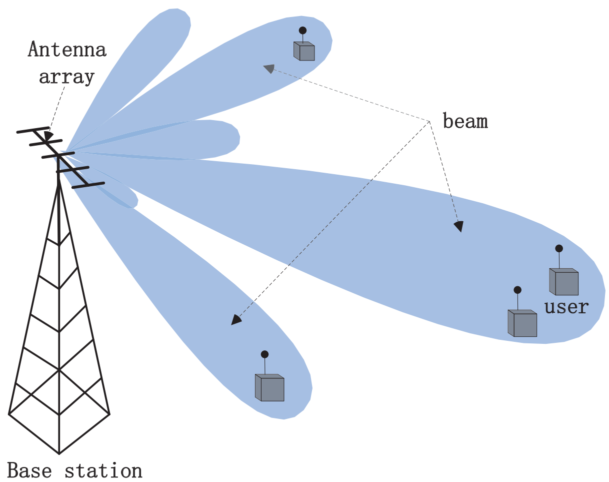

Fig. 1. System model. The base station equips antenna array. The shadows are the transmitting beams pointed to different users or users group.

For simplicity, we assume the network coded packets are modulated by BPSK, and the wireless channels obey square-law path loss with Rayleigh fading distribution. P_k is set to be the transmitting power of beam pointed to user k. According to the link budget relationship [6] and transmitting beamforming [12], the received SNR γ_k at k can be written as

$$\gamma_k = \frac{G_{beam} G P_k |h|^2}{d_k^2 N_0 R_b}, \tag{1}$$

where $|h|$ is the wireless channel gain with Rayleigh distribution; G_{beam} is the beamforming gain[2] that is decided by the weight vector on antenna elements of array [12]; d_k is the distance between BS and user k; N_0 is the white Gaussian noise power at the receiver and R_b indicates the bit transmission rate. Note that $G = (G_r G_t \lambda^2)/(M_l N_f (4\pi)^2)$ [6], where G_t and G_r are the transmitter and receiver antenna gains respectively, λ is the carrier wavelength, M_l designates the link margin that compensates for the hardware process variations and N_f denotes the receiver noise figure.

Given P_k and d_k, the bit error rate of k can be expressed as $p_k^b(P_k) = 0.5\left(1 - \sqrt{\frac{G_{beam} G P_k}{d_k^2 N_0 R_b} / (1 + \frac{G_{beam} G P_k}{d_k^2 N_0 R_b})}\right)$[7] . For the convenience of analysis, we do not use any error correcting code. Therefore, the PER p_k^p can be expressed as [8]:

$$p_k^p(P_k) = 1 - (1 - p_k^b(P_k))^N, \tag{2}$$

where N denotes the packet bit length. After receiving the coded packet, each user sends one-bit feedback messages[3] to inform the BS whether the packet has been received successfully or not. Here we assume the BS has L HDCP data associated with a deadline of T_s time slots. User k achieves a throughput $\Gamma_k = l \le L$ if it can recover l consecutive original data by T_s. The average network throughput per time unit across K users is defined as

$$\Gamma = \mathbb{E}\left(\frac{\sum_{k=1}^{K} \Gamma_k}{K T_s}\right), \tag{3}$$

where $\mathbb{E}[\cdot]$ is the expected function. The focus of this letter is to design a cross-layer transmission scheme to maximize (3) over the fading channels.

III. Joint dynamic Beamforming and Random Network Coding (DBRNC)

During the transmission, at the beginning of each time slot, the BS receives one-bit feedback information from each user to indicate whether or not the previously transmitted packet has been successfully received. Based on the feedback information, the BS decides which action to take at each time slot. The network dynamics can be modeled as a MDP. In particular, we specify the network dynamics by $(\mathbf{A}, \mathbf{S}, \mathbf{P}, r, T_s)$, in which boldface letters refer to vectors or matrices.

1) T_s is the number of time slots associated with the deadline. The time slot index is t $(0 \le t \le T_s - 1)$.
2) The action set $\mathbf{A}$: It consists of actions $a \in \mathbf{A}$ which send the network coded packets generated from different generations(one action corresponds to one time slot). For example, by taking the action $a^t = \{\textit{transmit } c_2 \textit{ from } G_2\}$, the BS transmits $c_2 = \beta_{12} a_1 + \beta_{22} a_2$ generated from G_2 to the network at time slot t.
3) Network state space $\mathbf{S}$: It is defined by a matrix

$$\mathbf{S} = \begin{bmatrix} \mathbf{s}_1 \\ \vdots \\ \mathbf{s}_k \\ \vdots \\ \mathbf{s}_K \end{bmatrix} = \begin{bmatrix} s_{1,1} & \dots & s_{1,l} & \dots & s_{1,L} \\ \vdots & \dots & \vdots & \dots & \vdots \\ s_{k,1} & \dots & s_{k,l} & \dots & s_{k,L} \\ \vdots & \vdots & \dots & \vdots & \\ s_{K,1} & \dots & s_{K,l} & \dots & s_{K,L} \end{bmatrix}, \tag{4}$$

where the element $\mathbf{s}_k = [s_{k,1}, \dots, s_{k,l}, \dots, s_{k,L}]$. Here $s_{k,l}$ is the number of network coded packet (generated from generation G_l) which is successfully received at user k. For example, if $K = 2$ and $L = 2$, at time slot t, the network state may be shown as $\mathbf{S}^t = \begin{bmatrix} 2 & 0 \\ 1 & 0 \end{bmatrix}$,which means user 1 received c_1 successfully twice and user 2 received c_1 once. 0 means both two users did not successfully received c_2.
4) Transmitting power allocation space $\mathbf{P}$: It reflects the transmitting power allocated to different beams, and can be expressed as

$$\mathbf{P} = [P_1, P_2, \dots, P_k, \dots, P_K], \Sigma_{k=1}^{K} P_k = P_c, \tag{5}$$

where P_k is the transmitting power of the beam pointing to user k and P_c is a constant representing the total transmitting power. Note that $\mathbf{P}^t$ indicates the status of power allocation at time slot t.
5) The immediate reward $r(\mathbf{S}^t, \mathbf{P}^t, a^t)$. This reward represents the benefit (i.e., the network throughput) associated with the current action a^t under given power allocation $\mathbf{P}^t$ and network state $\mathbf{S}^t$ at time slot t. It can be calculated as

$$\begin{aligned} r(\mathbf{S}^t, \mathbf{P}^t, a^t) &= \mathbb{E}\left[r(\mathbf{S}^{t+1} | \mathbf{S}^t, \mathbf{P}^t, a^t)\right] \\ &= \sum_{k=1}^{K} \mathbb{E}\left[r(\mathbf{s}_k^{t+1} | \mathbf{s}_k^t, P_k^t, a^t)\right]. \end{aligned} \tag{6}$$

[2]The number of beams and the gain of each beam can be achieved by adjusting the weight vector on the antenna elements. Due to space limit, we focus on the final beamforming gain of each beam and details on how to design the weight vector to generate beams are omitted (see[5],[11], [12]and references therein).

[3]The feedback information is transmitted via control channel and we assume it is error-free because of its short length (one bit).

where $\mathbb{E}[\cdot]$ is the function expected to $\mathbf{S}^{t+1}$; P_k^t is the power of the beam pointing to user k at time slot t and $\mathbf{P}^t = [P_1^t, P_2^t, \ldots, P_k^t, \ldots, P_K^t]$; vector $\mathbf{s}_k^t$ and $\mathbf{s}_k^{t+1}$ represent the network state of user k at time slot t and $t+1$; $r(\mathbf{s}_k^{t+1}|\mathbf{s}_k^t, P_k^t, a^t)$ is the future-dependent reward function of user k from $\mathbf{s}_k^t$ to $\mathbf{s}_k^{t+1}$ under given P_k^t and a^t. Note that, conditioning on $\mathbf{s}_k^t$, $\mathbf{s}_k^{t+1}$ only has two states: receive coded packet successfully (denoted as $\mathbf{s}_k^{t+1}(1)$) or not (denoted as $\mathbf{s}_k^{t+1}(0)$). For example, in the case of $L=4$, if the network state of user k at time slot t is $\mathbf{s}_k^t = [\ 1 \quad 1 \quad 0 \quad 0\ ]$, and the BS takes action $a^t =$ "sending a coded packet c_3 generated from G_3", the probability for next state $\mathbf{s}_k^{t+1}(1) = [\ 1 \quad 1 \quad 1 \quad 0\ ]$ is $p(\mathbf{s}_k^{t+1}(1)|\mathbf{s}_k^t, P_k^t, a^t) = 1 - p_k^p(P_k^t)$ and the reward (the number of successfully decoded original packet) $r(\mathbf{s}_k^{t+1}(1)|\mathbf{s}_k^t, P_k^t, a^t) = 3$. Thus, we can rewrite (6) as

$$r(\mathbf{S}^t, \mathbf{P}^t, a^t) = \sum_{k=1}^{K}\sum_{i=0}^{1}\left(1 - p_k^p(P_k^t)\right)^i \left(p_k^p(P_k^t)\right)^{(1-i)} r(\mathbf{s}_k^{t+1}(i)|\mathbf{s}_k^t, a^t) \tag{7}$$

This reward function is designed to show whether the action decided by the BS can bring higher network benefit, in other words, whether the receivers are able to decode the HDCP data as much as possible.

Once the network parameters are specified for each time slot, we seek for transmission policy that combines power allocation among antenna beams (at physical layer) and RNC encoding and scheduling (at network layer). A transmission policy $\mathbf{\Omega}$ is specified by $\mathbf{P}^t \in \mathbf{P}$ and $a^t \in \mathbf{A}$ in each time slot t. Let a non-negative real value function $\Gamma_{\mathbf{\Omega}}$ represent the expected reward obtained by following policy $\mathbf{\Omega}$. Assume that at the beginning the system is in state $\mathbf{S}^0$, the expected reward under policy $\mathbf{\Omega}$ is defined as

$$\Gamma_{\mathbf{\Omega}}(\mathbf{S}) = \mathbb{E}\left[\frac{1}{KT_s}\left(\sum_{t=0}^{T_s-1} r(\mathbf{S}^t, \mathbf{P}^t, a^t) + r(\mathbf{S}^{T_s})\right)\right], \tag{8}$$

where $\mathbf{\Omega} = [(\mathbf{P}^0, a^0), \ldots, (\mathbf{P}^t, a^t), \ldots, (\mathbf{P}^{T_s-1}, a^{T_s-1})]$. Our goal is to find an optimal policy $\mathbf{\Omega}^*$ in T_s steps that maximizes the expected cumulative reward (average network throughput):

$$\mathbf{\Omega}^* = \arg\max_{\mathbf{\Omega}} \{\Gamma_{\mathbf{\Omega}}(\mathbf{S})\} \tag{9}$$

Apparently, the current decision $(\mathbf{P}^t, a^t)$ is based on the current state and next state, which enforces $(\mathbf{P}^t, a^t)$ to be adjusted dynamically during the transmission process. Meanwhile, the decision in the current time slot also affects the network state in the next time slot. Under this cross-layer design structure, we adopt a low-complexity greedy scheduling technique (GST) to solve optimization problem (9). In GST, the BS finds the appropriate $(\mathbf{P}^t, a^t)$ that maximizes the network throughput in each time slot. In particular, based on the feedback information from the users, the BS updates the network state and selects suitable $\mathbf{P}^t$ and a^t in each time slot to maximize the one time step future-reward. Combining with (6), we get

$$\begin{aligned}(\mathbf{P}^t, a^t)^* &= \arg\max_{a^t \in \mathbf{A}}(\max_{\mathbf{P}^t \in \mathbf{P}} r(\mathbf{S}^t, \mathbf{P}^t, a^t)); \\ &\text{subject to } \| \mathbf{P}^t \| = P_c.\end{aligned} \tag{10}$$

For each a^t, in order to find the optimal $\mathbf{P}^t$ with the maximum $r(\mathbf{S}^t, \mathbf{P}^t, a^t)$, we construct Lagrange multiplier function shown in (11). Taking the partial derivative of $\Lambda(\mathbf{P}^t, \lambda)$ with respect to $\mathbf{P}^t$ and λ, we get (12). Fortunately, P_k^ts are decoupled with each other in (12). Furthermore, it is easy to verify that $p_k^p(P_k^t)$ is concave with respect to P_k^t. Therefore, we can use function "fmincon" in Matlab or Lagrange multiplier optimal algorithm [9] to get the optimal $\mathbf{P}^t$ with lower complexity.

Finally, by comparing the maximum reward $r(\mathbf{S}^t, \mathbf{P}^t, a^t)$ for all possible action a^t, we arrive at the optimal tuple $(\mathbf{P}^t, a^t)^*$ that maximizes the overall reward of each time slot. Guided by $(\mathbf{P}^t, a^t)^*$, the BS makes a transmission decision and moves to the next state $\mathbf{S}^{t+1}$. This process will go on until the deadline. After T_s time slots, we can get the optimal $\mathbf{\Omega}^* = [(\mathbf{P}^0, a^0)^*, \ldots, (\mathbf{P}^t, a^t)^*, \ldots, (\mathbf{P}^{T_s-1}, a^{T_s-1})^*]$.

The pseudocode of DBRNC is shown in Algorithm 1.

Algorithm 1 : DBRNC algorithm using GST technique

Input: $\mathbf{A}, \mathbf{P}, T_s, L, K$.
Output: $\mathbf{\Omega}^* = [(\mathbf{P}^0, a_0)^*, \ldots, (\mathbf{P}^t, a^t)^*, \ldots, (\mathbf{P}^{T_s-1}, a^{T_s-1})^*]$

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Initialize: S^0 = [0] and r(S^0) = 0
for t = 0 to T_s − 1 do
    for each a^t ∈ A do
        Γ*(P^t, a^t) = max_{P^t∈P} r(S^t, P^t, a^t)
    end for
    (P^t, a^t)* = arg max_{a^t∈A} Γ*(P^t, a^t)
end for
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Remark 1: During the MDP, $\mathbf{P}$ is adjusted dynamically according to the current network state $\mathbf{S}$ and the action a^t. On the other hand, $\mathbf{A}$ and $\mathbf{P}$ together determine the next network state. This fully reflects the cross-layer design structure.

IV. Simulation and Discussions

In simulation, we prioritize the data into four interdependent layers ($L=4$) in the decreasing order of importance $\alpha_1 \geq \alpha_2 \geq \alpha_3 \geq \alpha_4$ [10], where each layer is assumed to be encapsulated into one HDCP data. At physical layer, the parameters described in Section II are given as follows: $T_s = 6$, $R_b = 10$kb/s, $G_{beam} = 7$dB, $N = 1$K, $f_c = 2$GHz, $G_tG_r = 5$dB, $N_f = 10$dB, $M_l = 40$dB, $N_0/2 = -174$dBm.

$$\Lambda(\mathbf{P}^t, \lambda) = \sum_{k=1}^{K}\sum_{i=0}^{1}\left(1 - p_k^p(P_k^t)\right)^{(1-i)}\left(p_k^p(P_k^t)\right)^i r(\mathbf{s}_k^{t+1}(1-i)|\mathbf{s}_k^t, a^t) + \lambda(\sum_{k=1}^{K} P_k - P_c). \tag{11}$$

$$\begin{cases}\frac{\partial\Lambda(\mathbf{P}^t,\lambda)}{\partial P_k^t} = \frac{\partial p_k^p(P_k^t)}{\partial P_k^t}\left(r(\mathbf{s}_k^{t+1}(0)|\mathbf{s}_k^t, a^t) - r(\mathbf{s}_k^{t+1}(1)|\mathbf{s}_k^t, a_t)\right) + r(\mathbf{s}_k^{t+1}(1)|\mathbf{s}_k^t, a^t) + \lambda = 0, 1 \leq k \leq K \\ \frac{\partial\Lambda(\mathbf{P}^t,\lambda)}{\partial\lambda} = \sum_{k=1}^{K} P_k - P_c = 0\end{cases} \tag{12}$$

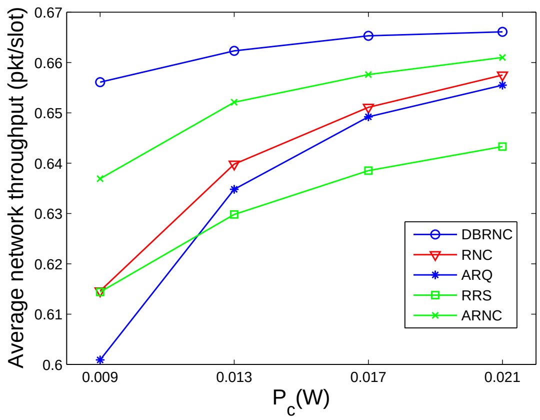

Fig. 2. Average network throughput vs. transmitting power

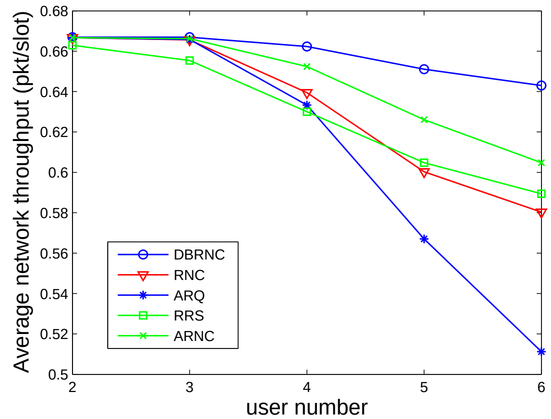

Fig. 3. Average network throughput vs. number of users

In Fig. 2, we investigate the impact of the total transmitting power P_c on the average network throughput Γ. As an example, we generate 4 antenna beams pointing to $K = 4$ users, whose distances to BS are 360m, 120m, 50m and 220m respectively. To illustrate the effectiveness of our algorithm, we compare with other algorithms including Automatic Repeat Request (ARQ), Round-Robin Scheduling (RRS), Random Network Coding (RNC), and Adaptive Random Network Coding (ARNC) [4]. For a fair comparison, we also use beamforming in the above schemes, and let each beam pointed to the user shares the same transmitting power but without cross-layer design. As expected, the Γ of each algorithm increases with P_c. This is because a higher P_c means a lower PER, which increases the network throughput. Furthermore, our DBRNC algorithm has the best performance among all algorithms. This is because we fully exploit the performance gains by dynamic beamforming at physical layer and RNC at network layer. Note that the higher throughput of DBRNC is achieved with reasonable computational complexity. Most importantly, we observe that the DBRNC has the biggest performance improvement in the low P_c region, which means DBRNC is also energy-efficient.

Fig. 3 depicts the average network throughput versus the number of users, where we fix $P_c = 1.3 \times 10^{-2}$W. The distances between the BS and users are set as 360m, 120m, 50m, 220m, 370m, 290m, respectively. Apparently our method offers the best performance by jointly employing beamforming and network coding. The performance gaps between DBRNC and other algorithms become larger as the number of receivers increases. This is because each DBRNC packet brings additional information to many receivers, alleviating the bottleneck effect. Meanwhile, the dynamic power allocation by beamforming is more advantageous for larger number of users.

V. Conclusion

In this letter we studied a new multicast transmission scheme that jointly combines RNC at network layer and smart antenna beamforming at physical layer to improve the network throughput. Within this cross-layer framework, we proposed the DBRNC algorithm that achieves both power gain by dynamic power allocation and coding gain by adaptive RNC encoding and scheduling. Simulation results have shown the proposed method has significant performance improvement over existing approaches.

Acknowledgment

The authors would like to thank XXX

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