

Letters

# Adaptive neural network control of uncertain nonlinear systems with nonsmooth actuator nonlinearities

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## Abstract

In this paper, we present a new approach of designing adaptive neural network controllers for uncertain systems containing nonsmooth nonlinearities in the actuator device. The controllers are designed by introducing certain well-defined sign functions and neural network approximations as well as by using the backstepping technique. The salient feature of the approach is that no knowledge is assumed on unknown system parameters and nonlinearities. It is shown that the proposed controller not only can guarantee semi-global stability, but also excellent transient performance can be achieved.

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## 1. Introduction

Adaptive control is becoming popular in many fields of engineering and science as concepts of adaptive systems are becoming more attractive in developing advanced applications. It faces many important challenges, especially in nontraditional applications, such as nonsmooth nonlinearities. Several adaptive control schemes have recently been proposed, see for examples [16,9,2,1]. In these papers, an adaptive inverse technique was constructed to cancel the effects of nonlinearities. The compensation scheme is considered in [16] for hysteresis, [17] for backlash, [8] for dead-zone and [15] for actuator failure. In [3], variable structure control was proposed to stabilize the nonlinear plants by using a quadratic and a dynamic description of the nonlinearity, where the parameters of the dead-zone and backlash nonlinearities are bounded by known constants. In [14,20,21] a dynamic dead-zone or backlash model was defined to pattern such nonlinearity rather than constructing an inverse model to mitigate the effect, where

the effect of dead-zone or backlash was treated as a bounded disturbance. In [22], a new smooth inverse was constructed to compensate the effects of dead-zone by using backstepping technique. In [11], a compensation scheme using two neural networks was developed for nonlinear actuator dead-zone.

This paper addresses control design for nonlinear systems with unknown parameters and nonsmooth nonlinearities in the actuator. The existence of such nonsmooth nonlinearities imposes a great challenge for the controller development. To address such a challenge, neural networks (NNs) will be adopted to model the plant and the controller is constructed based on NNs. The NNs, used to approximate nonlinearities in the plant, are adjusted by an adaptive law based on the backstepping approach. To compensate for the effects from the NN approximation, we propose a signal function, which is continuous and differentiable, and employ it in the recursive backstepping technique. The estimators are used to handle such terms. By virtue of the approximation of nonsmooth nonlinearities, the new function and the backstepping technique, a priori knowledge on system parameters and nonlinearities is no longer needed. Besides showing stability of the

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system, transient performance in terms of  $L_2$  norm of the tracking error is derived. It turns out that the tracking error is an explicit function of design parameters and thus the proposed scheme allows designers to achieve closed-loop behavior by tuning design parameters in an explicit way.

**2. Problem statement**

Consider a single-input single-output nonlinear system of the following form:

$$\dot{x}_i(t) = x_{i+1}(t) + \theta^T \phi_i(\bar{x}_i(t)), \quad i = 1, \dots, n - 1,$$

$$\dot{x}_n(t) = \phi_0(x(t)) + \theta^T \phi_n(x(t)) + \omega, \tag{1}$$

$$\omega = N(u), \quad y(t) = x_1(t), \tag{2}$$

where  $\bar{x}_i(t) = [x_1(t), \dots, x_i(t)]^T$ ,  $x(t) = [x_1(t), \dots, x_n(t)]^T \in R^n$ ,  $\omega \in R$  and  $y(t)$  are state variables, system input and output, respectively,  $\theta \in R^r$  is an unknown constant parameter vector and  $\phi_0(\cdot) \in R$  and  $\phi_i(\cdot) \in R^r, i = 1, \dots, n$ , are known smooth nonlinear functions. The nonlinear system is assumed to be preceded by the actuating device  $\omega = N(u)$  (see Fig. 1),  $\omega$  being the actuator output not available for control and  $u$  being the actuator input. It should be noted that more general classes of nonlinear systems can be transformed into this structure [18,19]. In this paper the following actuator nonsmooth nonlinearities have been considered to be present in the actuator:

*Dead-Zone:* The analytical expression of the dead-zone characteristic is in [18].

$$\omega(t) = N(u(t)) = \begin{cases} m_r(u(t) - b_r), & u(t) \geq b_r, \\ 0, & b_l < u(t) < b_r, \\ m_l(u(t) - b_l), & u(t) \leq b_l, \end{cases} \tag{3}$$

where  $b_r \geq 0, b_l \leq 0$  and  $m_r > 0, m_l > 0$  are constants. In general, the break points  $|b_r| \neq |b_l|$  and the slopes  $m_r \neq m_l$ .

*Backlash:* The backlash nonlinearity is described by

$$\omega(t) = N(u(t)) = \begin{cases} m(u(t) - B_r) & \text{if } \dot{\omega}(t) > 0 \text{ and } \omega(t) = m(u(t) - B_r), \\ m(u(t) - B_l) & \text{if } \dot{\omega}(t) < 0 \text{ and } \omega(t) = m(u(t) - B_l), \\ \omega(t_-) & \text{otherwise,} \end{cases} \tag{4}$$

where  $m \geq m_0$  is the slope of the lines, with  $m_0$  being a small positive constant, and  $B_r > 0, B_l < 0$  are constant parameters,  $u(t_-)$  means no change occurs in the output control signal  $u(t)$ .

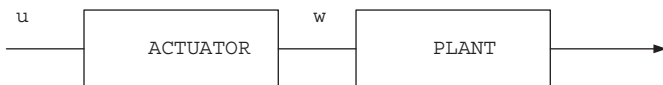


Fig. 1. Block scheme of a plant driven by the actuator.

The idea pursued in this paper is to design backstepping neural network control laws that are able to achieve robust performances in the presence of the above nonsmooth nonlinearities with uncertainties.

**Assumption 1.** The desired trajectory  $y_r(t)$  and its  $(n - 1)$ th order derivatives are known and bounded in a compact set  $\Omega_r$ .

The control objectives are to design an adaptive control law such that

- the closed-loop system is stable in the sense that all the signals in the loop are bounded;
- the tracking error  $y(t) - y_r(t)$  is adjustable during the transient period by an explicit choice of design parameters and  $\lim_{t \rightarrow \infty} |y(t) - y_r(t)| \leq \delta_1$  for an arbitrary specified bound  $\delta_1$ .

**3. Function approximation using neural networks**

In this section, we present NN approximation of a piecewise continuous function as in [11,5]. It is found that to approximate such functions suitably, it is necessary to augment the standard NN that uses smooth activation functions with extra nodes containing a certain jump function approximation basis set of activation functions. For the neural networks, the capability of a NN in approximating functions with a specified degree of accuracy has been proven and demonstrated in [19].

*3.1. Neural network approximation of continuous functions*

Any functions can be approximated by a single-hidden layer feedforward network (SLFN) mapping with appropriate weights on a compact set. Note that the idea of using two layer neural network in which the hidden layer performs a fixed nonlinear transformation with no adjustable parameters was suggested in [11]. In other words, any function  $f(x) \in C(S)$ , with  $S$  a compact subset of  $R^n$ , there exists

$$f(x) = \sum_{k=0}^L w_k \sigma_k(m_k^T x + n_k) + \varepsilon(x) = W^T \sigma(M, x, N) + \varepsilon(x), \tag{5}$$

where the weights  $w_k \in R (k = 0, \dots, L)$  and  $m_k \in R^n (k = 0, \dots, L)$  are vectors,  $x \in R^n, W = [w_0, \dots, w_L]^T, M = [m_0, \dots, m_L]^T, N = [n_0, \dots, n_L]^T, \sigma(\cdot) = [\sigma_0(\cdot), \dots, \sigma_L(\cdot)]^T$  and  $\varepsilon(x)$  is the NN approximation error, which is bounded by  $\|\varepsilon(x)\| < \varepsilon_N$  for all  $x \in S, \varepsilon_N$  is a positive constant. The weights  $m_k$  and  $n_k$  in the first layer are selected randomly and will not be tuned. The weights  $w_k$  in the second layer are tunable. The function  $\sigma(\cdot)$  could be any continuous sigmoid function. Here, we choose  $\sigma(\cdot)$  as

$$\sigma(t) = \frac{1}{1 + \exp^{-t}}. \tag{6}$$

This result shows that any continuous function can be approximated arbitrarily well using a linear combination of sigmoidal functions. This is well known as the NN universal approximation property. Recently, a novel learning method for SLFN named extreme learning machine (ELM) algorithm has been proposed in [6,7]. Its universal approximation capability has been proved in [5]. In our algorithm, the input weights and the bias of the hidden neurons are randomly generated and kept fixed as in ELM. The output weights will be determined by adaptive laws based on Lyapunovs stability theory.

### 3.2. Compensation of nonlinearities

In this section, an NN precompensator for a general model is given. The generality of the method and applicability to a broad range of nonlinear functions make this approach a potentially useful tool for compensation of backlash, hysteresis, and other nonlinearities.

For any unknown nonlinear function  $N(u)$ , we have the following assumption.

**Assumption 2.** The function  $N(u)$  is invertible and continuous.

By assumption, there exists  $N^{-1}(v)$ , such that

$$N(N^{-1}(v)) = v. \tag{7}$$

The function  $N^{-1}(v)$  can be expressed in the following equivalent form:

$$N^{-1}(v) = v + \omega_{NN}(v), \tag{8}$$

where  $\omega_{NN}(v) = N^{-1}(v) - v$ . Eq. (8) can be viewed as a direct feedforward term plus a correction term.

Based on the NN approximation property, one can approximate the nonlinear function by

$$N(u) = W^T \sigma(M, u, N) + \varepsilon(u). \tag{9}$$

Also, we can design an NN for the approximation of the modified inverse function given in (8) by

$$\omega_{NN}(v) = \hat{W}_0^T \sigma(M_0, v, N_0) + \varepsilon_0(v). \tag{10}$$

In these equations,  $\varepsilon(u), \varepsilon_0(v)$  are the NN reconstruction errors and  $W, W_0$  are ideal target weights. The reconstruction error is bounded by  $\|\varepsilon(u)\| < \varepsilon_N$  and  $\|\varepsilon_0(v)\| < \varepsilon_{N_0}$ , where  $\varepsilon_N$  and  $\varepsilon_{N_0}$  are positive constants. The weights in the first layer  $M, M_0, N, N_0$  in both (9) and (10) are fixed.

Define  $\hat{W}, \hat{W}_0$  as estimates of the ideal NN weights  $W, W_0$  and define the weight estimation errors as

$$\tilde{W} = W - \hat{W}, \quad \tilde{W}_0 = W_0 - \hat{W}_0. \tag{11}$$

It follows that the estimators of nonlinearity  $N(u)$  and modified inverse can be approximated by and estimations of the nonlinearity and modified inverse function as

$$\hat{N}(u) = \hat{W}^T \sigma(M, u, N), \tag{12}$$

$$\hat{\omega}_{NN}(v) = \hat{W}_0^T \sigma(M_0, v, N_0). \tag{13}$$

Note that

$$u = v + \hat{\omega}_{NN}(v). \tag{14}$$

Two NNs are exploited, the first NN is used as an estimator of nonlinearity, while the second is used as a compensator. The structure of the NN precompensator and estimator is shown in Fig. 2.

**Theorem 1.** Given the NN observer (12) and the NN approximation functions (13), (14), the approximation of  $N(u)$  is given by

$$N(u) = v - \tilde{W}^T \sigma(M, u, N) M^T \tilde{W}_0^T \sigma(M_0, v, N_0) + \tilde{W}^T \sigma(M, u, N) M^T \hat{\omega}_{NN} + d(t), \tag{15}$$

where the modelling mismatch term  $d(t)$  is given by

$$d(t) = -\tilde{W}^T \sigma'(M, u, N) M^T W_0^T \cdot \sigma(M_0, v, N_0) - b(t) + \varepsilon(u), \tag{16}$$

$$b(t) = W^T \sigma'[M, v + \hat{W}_0^T \sigma(M_0, v, N_0), N] \cdot M^T \varepsilon_0(v) + W^T R_1(\tilde{W}_0, v) + \varepsilon(v + \omega_{NN}), \tag{17}$$

where  $R_1(\cdot)$  is the remainder of the first Taylor polynomial.

**Proof.** The proof is similar to [11]. From (7) to (10)

$$\begin{aligned} v &= N(v + \omega_{NN}(v)) \\ &= W^T \sigma[M^T, (v + \hat{W}_0^T \sigma(M_0, v, N_0) \\ &\quad + \tilde{W}_0^T \sigma(M_0, v, N_0) + \varepsilon_0(v)), N] + \varepsilon(v + \omega_{NN}(v)). \end{aligned} \tag{18}$$

Using the Taylor series expansion, we have

$$\begin{aligned} v &= W^T \sigma[M^T, (v + \hat{W}_0^T \sigma(M_0, v, N_0) + N) \\ &\quad + W^T \sigma'[M^T, (v + \hat{W}_0^T \sigma(M_0, v, N_0)), N] \\ &\quad \cdot V^T (\tilde{W}_0^T \sigma(M_0, v, N_0) + b(t), \end{aligned} \tag{19}$$

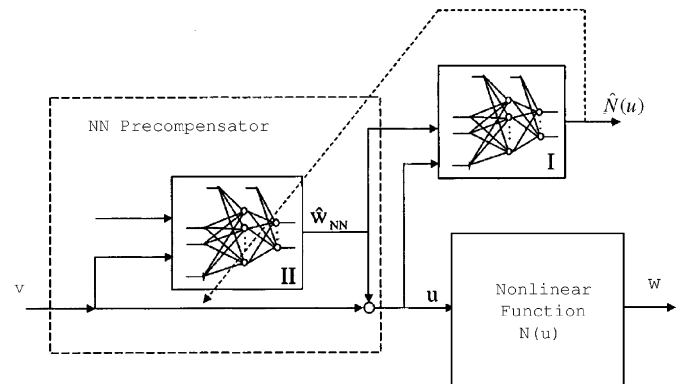


Fig. 2. NN compensation scheme.

where  $b(t)$  is defined in (17). Then from (14) and (19), we have

$$\begin{aligned}
 v + \varepsilon(u) &= W^T \sigma[M, u, N] + \varepsilon(u) + \hat{W}^T \sigma'[M, u, N] \\
 &\quad \cdot M^T (\hat{W}_0^T \sigma(M_0, v, N_0)) + \tilde{W}^T \sigma'[M, u, N] \\
 &\quad \cdot M^T (W_0^T \sigma(M_0, v, N_0)) - \tilde{W}^T \sigma'[M, u, N] \\
 &\quad \cdot M^T (\hat{W}_0^T \sigma(M_0, v, N_0)) + b(t). \tag{20}
 \end{aligned}$$

Combining with  $N(u) = W^T \sigma(M, u, N) + \varepsilon(u)$  in (9) and  $\hat{\omega}_{NN}(v) = \hat{W}_0^T \sigma(M_0, v, N_0)$  in (13), we can get (15). In (15), the first term has known factors multiplying  $\tilde{W}$ , while the second term has known factors multiplying  $\tilde{W}_0$  and a suitable bound term can be found for  $d(t)$ .  $\square$

Theorem 1 shows the effectiveness of the proposed NN structure. It shows that the estimates  $\hat{W}$ ,  $\hat{W}_0$  approach the actual neural network parameters  $W$ ,  $W_0$ , and the NN effectively provides a pre-inverse for the nonlinearity. It will be shown that closed-loop stability can be guaranteed in deriving the NN update laws for  $\hat{W}$ ,  $\hat{W}_0$ .

**Remark 1.** Note that the form of (15) is crucial in controller design and in deriving adaptive laws that guarantee closed-loop stability. Moreover, the residual term is bounded by a constant vector multiplied by a known function vector as in (21). Thus, adaptive control techniques can be applied to deal with this residual term. A similar technique has also been used in [11,19,12], where the approximator was constructed by neural networks or fuzzy logic. In general, this neural network scheme or fuzzy logic scheme could be used for any continuous invertible functions. Therefore, it is a very significant result concerning general nonlinearities in motion control systems.

The following result gives the upper bound of the norm  $d(t)$ , which was proposed in [11]. This is an important result to be used in the stability proof, where  $\|\cdot\|$  as any suitable vector norm.

**Lemma 1.** *The norm of the modelling mismatching term  $d(t)$  in (15) is bounded by*

$$\|d(t)\| \leq \beta^T Y(t), \tag{21}$$

where  $\beta \in \mathbb{R}^{4 \times 1}$  is an unknown constant vector, being composed of optimal weight matrices and some bounded constants and  $Y(t) = [1, \|\hat{W}\|, \|\hat{W}_0\|^2, \|\hat{W}_0\|]^T$  is a known function vector.

**Proof.** The proof is similar to [11]. From (16), it has

$$\begin{aligned}
 \|d(t)\| &\leq \|\tilde{W}\| \|\sigma'(\cdot)\| \|M\| \|W_0\| \|\sigma_0(\cdot)\| \\
 &\quad + \|b(t)\| + \|\varepsilon(v + \hat{\omega}_{NN})\|, \tag{22}
 \end{aligned}$$

where  $\sigma_0(\cdot) = \sigma(M_0^T v + N_0)$ . It is obvious that there should exist positive constants  $\bar{W}$  and  $\bar{W}_0$  satisfying  $\|W\| \leq \bar{W}$  and  $\|W_0\| \leq \bar{W}_0$ , where  $\bar{W}$  and  $\bar{W}_0$  are not needed to be known. Based on the facts

$$\|\tilde{W}\| \leq \|W\| + \|\hat{W}\| \leq \bar{W} + \|\hat{W}\|, \tag{23}$$

$$\|\tilde{W}_0\| \leq \|W_0\| + \|\hat{W}_0\| \leq \bar{W}_0 + \|\hat{W}_0\|, \tag{24}$$

we have

$$\begin{aligned}
 \|d(t)\| &\leq \|\tilde{W}\| \|\sigma'(\cdot)\| \|M\| \bar{W}_0 \|\sigma_0(\cdot)\| + \|b(t)\| + \varepsilon_N \\
 &= a_1 \|\tilde{W}\| + \|b(t)\| + \varepsilon_N \\
 &\leq a_1 \|\hat{W}\| + a_1 \bar{W} + \|b(t)\| + \varepsilon_N, \tag{25}
 \end{aligned}$$

where  $a_1 = \|\sigma'(\cdot)\| \|M\| \bar{W}_0 \|\sigma_0(\cdot)\|$ . From definition (17) follows

$$\begin{aligned}
 \|b(t)\| &\leq \|W\| \|\sigma'(\cdot)\| \|M\| \|\varepsilon_0(v)\| \\
 &\quad + \|W\| \|\sigma''(\cdot)\| \|M\|^2 \|\tilde{W}_0\|^2 \|\sigma_0(\cdot)\|^2 \\
 &\quad + \|W\| \|\sigma''(\cdot)\| \|M\|^2 \|\tilde{W}_0\| \|\sigma_0(\cdot)\| \|\varepsilon_0(v)\| \\
 &\quad + \|\varepsilon(v + \omega_{NN})\| + \|W\| \|\sigma''(\cdot)\| \|M\|^2 \|\varepsilon_0(v)\|^2 \\
 &\leq \bar{W} \|\sigma'(\cdot)\| \|M\| \varepsilon_{N0} + \bar{W} \|\sigma''(\cdot)\| \\
 &\quad \times \|M\|^2 \|\tilde{W}_0\|^2 \|\sigma_0(\cdot)\|^2 \\
 &\quad + \bar{W} \|\sigma''(\cdot)\| \|M\|^2 \|\tilde{W}_0\| \|\sigma_0(\cdot)\| \varepsilon_{N0} \\
 &\quad + \bar{W} \|\sigma''(\cdot)\| \|M\|^2 \varepsilon_{N0}^2 + \varepsilon_N \\
 &= a_2 \|\tilde{W}_0\|^2 + a_3 \|\tilde{W}_0\| + a_4 \\
 &\leq a_2 \|\hat{W}_0\|^2 + a_3 \|\hat{W}_0\| + a_2 \bar{W}_0^2 + a_s \bar{W}_0 + a_4, \tag{26}
 \end{aligned}$$

where  $a_2 = \bar{W} \|\sigma''(\cdot)\| \|M\|^2 \|\sigma_0(\cdot)\|^2$ ,  $a_3 = \bar{W} \|\sigma''(\cdot)\| \|M\|^2 \|\sigma_0(\cdot)\| \varepsilon_{N0}$ ,  $a_4 = \bar{W} \|\sigma'(\cdot)\| \|M\| \varepsilon_{N0} + \bar{W} \|\sigma''(\cdot)\| \|M\|^2 \varepsilon_{N0}^2 + \varepsilon_N$  are unknown constants. Thus from (25) and (26), we get

$$\begin{aligned}
 \|d(t)\| &\leq a_1 \|\hat{W}\| + a_1 \bar{W} + \varepsilon_N + a_2 \|\hat{W}_0\|^2 \\
 &\quad + a_3 \|\hat{W}_0\| + a_2 \bar{W}_0^2 + a_3 \bar{W}_0 + a_4 \\
 &= \beta^T \cdot [1, \|\hat{W}\|, \|\hat{W}_0\|^2, \|\hat{W}_0\|]^T = \beta^T Y, \tag{27}
 \end{aligned}$$

where  $\beta = [a_0, a_1, a_2, a_3]^T$ ,  $a_0 = a_1 \bar{W} + a_2 \bar{W}_0^2 + a_3 \bar{W}_0 + a_4 + \varepsilon_N$ .  $\square$

**Remark 2.** Note that  $\beta$  is not assumed to be known. So, the residual term  $d(t)$  is bounded by an unknown parameter vector with a known function vector as in (21). Unlike the normal NN approximation using the restricted assumption, the residual term is bounded by a known bound. All these uncertain parameter vectors will be estimated by the proposed adaptive update law.

**Remark 3.** The NN compensation scheme with Theorem 1 and Lemma 1 were also proposed in [11].

#### 4. Design of adaptive controllers

Before presenting the design of an adaptive controller using the backstepping technique to achieve the desired control objectives, the following change of coordinates is necessary:

$$z_1 = y - y_r, \tag{28}$$

$$z_i = x_i - y_r^{(i-1)} - \alpha_{i-1}, \quad i = 2, 3, \dots, n, \tag{29}$$

where  $\alpha_{i-1}$  is the virtual control at the  $i$ th step and will be determined in later discussions. We define functions  $sg_i(z_i)$

and  $\eta_i(z_i)$  as follows:

$$sg_i(z_i) = \begin{cases} \frac{z_i}{|z_i|}, & |z_i| \geq \delta_i, \\ \frac{2[2(n-i)+1]!}{(2\delta_i)^{2(n-i)+1}[(n-i)!]^2} \\ \quad \times \int_{-z_i}^{z_i} (\delta_i^2 - \chi^2)^{n-i} d\chi - 1, & |z_i| < \delta_i, \end{cases} \quad (30)$$

where  $\delta_i (i = 1, \dots, n)$  is a positive design parameter.

**Remark 4.** Note that  $sg_i(z_i)$  is  $(n-i+1)$ th order differentiable. The standard sign function is non-smooth and not differentiable, which cannot be used in the recursive backstepping design. Because backstepping requires all the signals are continuous and differentiated.

We also design a function  $\eta_i(z_i)$  as

$$\eta_i(z_i) = \begin{cases} 1, & |z_i| \geq \delta_i, \\ 0, & |z_i| < \delta_i. \end{cases} \quad (31)$$

Then we can get

$$sg_i(z_i)\eta_i(z_i) = \begin{cases} 1, & z_i \geq \delta_i, \\ 0, & |z_i| < \delta_i, \\ -1, & z_i \leq -\delta_i. \end{cases} \quad (32)$$

This ensures that the resulting functions are differentiable. We now illustrate the backstepping design procedure with details given for the last step.

- *Step 1:* It follows for (1) and (28) that

$$\dot{z}_1 = x_2 + \theta^T \phi_1 - \dot{y}_r = z_2 + \alpha_1 + \theta^T \phi_1. \quad (33)$$

We design virtual control law  $\alpha_1$  as

$$\alpha_1 = -(c_1 + \frac{1}{4})(|z_1| - \delta_1)^n sg_1(z_1) - \hat{\theta}^T \phi_1 - (\delta_2 + 1)sg_1(z_1), \quad (34)$$

where  $c_1$  is a positive design parameter. We choose Lyapunov function  $V_1$  as

$$V_1 = \frac{1}{n+1}(|z_1| - \delta_1)^{n+1} \eta_1 + \frac{1}{2} \tilde{\theta}^T \Gamma^{-1} \tilde{\theta}. \quad (35)$$

Using (33) and (34), the derivative of  $V_1$  is

$$\begin{aligned} \dot{V}_1 &= (|z_1| - \delta_1)^n \eta_1 sg_1(z_1) \dot{z}_1 + \tilde{\theta}^T \Gamma^{-1} \dot{\tilde{\theta}} \\ &\leq -(c_1 + \frac{1}{4})(|z_1| - \delta_1)^{2n} \eta_1 \\ &\quad + (|z_1| - \delta_1)^n (|z_2| - \delta_2 - 1) \eta_1 \\ &\quad + (|z_1| - \delta_1)^n \eta_1 sg_1(z_1) \tilde{\theta}^T \phi_1 - \tilde{\theta}^T \Gamma^{-1} \dot{\tilde{\theta}} \\ &\leq -(c_1 + \frac{1}{4})(|z_1| - \delta_1)^{2n} \eta_1 + (|z_1| - \delta_1)^n (|z_2| \\ &\quad - \delta_2 - 1) \eta_1 + \tilde{\theta}^T (\tau_1 - \Gamma^{-1} \dot{\tilde{\theta}}), \end{aligned} \quad (36)$$

$$\tau_1 = \phi_1 (|z_1| - \delta_1)^n \eta_1 sg_1(z_1). \quad (37)$$

- *Step 2:* Now, we evaluate the second state  $z_2$ . Differentiating (29) for  $i = 2$ , we have

$$\begin{aligned} \dot{z}_2 &= z_3 + \alpha_2 + \theta^T \phi_2 - \frac{\partial \alpha_1}{\partial x_1} (x_2 + \theta^T \phi_1) \\ &\quad - \frac{\partial \alpha_1}{\partial y_r} \dot{y}_r - \frac{\partial \alpha_1}{\partial \hat{\theta}} \dot{\hat{\theta}}. \end{aligned} \quad (38)$$

Note that  $\alpha_1$  is a function of  $x_1, y_r, \hat{\theta}$ . We design virtual control law  $\alpha_2$  as

$$\begin{aligned} \alpha_2 &= -\left(c_2 + \frac{5}{4}\right)(|z_2| - \delta_2)^{n-1} sg_2(z_2) \\ &\quad - (\delta_3 + 1)sg_2(z_2) + \frac{\partial \alpha_1}{\partial x_1} x_2 + \frac{\partial \alpha_1}{\partial y_r} \dot{y}_r + \frac{\partial \alpha_1}{\partial \hat{\theta}} \Gamma \tau_2 \\ &\quad - \hat{\theta}^T \left(\phi_2 - \frac{\partial \alpha_1}{\partial x_1} \phi_1\right), \end{aligned} \quad (39)$$

$$\tau_2 = \tau_1 + \left(\phi_2 - \frac{\partial \alpha_1}{\partial x_1} \phi_1\right) (|z_2| - \delta_2)^{n-1} \eta_2 sg_2(z_2),$$

where  $c_2$  is positive design parameter. We design Lyapunov function  $V_2 = (1/n)(|z_2| - \delta_2)^n \eta_2 + V_1$ . Then the derivative of  $V_2$  is

$$\begin{aligned} \dot{V}_2 &\leq -\sum_{i=1}^2 (|z_i| - \delta_i)^{2(n-1)} \eta_i \\ &\quad + M_2 + (|z_2| - \delta_2)^{n-1} (|z_3| - \delta_3 - 1) \eta_2 \\ &\quad + (|z_2| - \delta_2)^{n-1} \eta_2 sg_2(z_2) \frac{\partial \alpha_1}{\partial \hat{\theta}} \\ &\quad \times (\Gamma \tau_2 - \dot{\hat{\theta}}) + \tilde{\theta}^T (\tau_2 - \Gamma^{-1} \dot{\tilde{\theta}}), \end{aligned} \quad (40)$$

where  $M_2 = -(1/4)(|z_1| - \delta_1)^{2n} \eta_1 + (|z_1| - \delta_1)^n (|z_2| - \delta_2 - 1) \eta_1 - (|z_2| - \delta_2)^{2(n-1)} \eta_2$ . Now we show that  $M_2 < 0$ . It is clear that  $M_2 \leq 0$  for  $|z_2| < \delta_2 + 1$ . For  $|z_2| \geq \delta_2 + 1$ ,

$$\begin{aligned} M_2 &\leq -\frac{1}{4}(|z_1| - \delta_1)^{2n} \eta_1 + \frac{1}{4}(|z_1| - \delta_1)^{2n} \eta_1^2 \\ &\quad + (|z_2| - \delta_2 - 1)^2 - (|z_2| - \delta_2)^{2(n-1)} \\ &\quad < (|z_2| - \delta_2)^2 - (|z_2| - \delta_2)^{2(n-1)} \\ &= (|z_2| - \delta_2)^2 (1 - (|z_2| - \delta_2)^{2(n-2)}) \leq 0. \end{aligned} \quad (41)$$

Then (40) is written as

$$\begin{aligned} \dot{V}_2 &\leq -\sum_{i=1}^2 c_i (|z_i| - \delta_i)^{2(n-i+1)} \eta_i + \tilde{\theta}^T (\tau_2 - \Gamma^{-1} \dot{\tilde{\theta}}) \\ &\quad + (|z_2| - \delta_2)^{n-1} (|z_3| - \delta_3 - 1) \eta_2 \\ &\quad + (|z_2| - \delta_2)^{n-1} \eta_2 sg_2(z_2) \frac{\partial \alpha_1}{\partial \hat{\theta}} (\Gamma \tau_2 - \dot{\hat{\theta}}). \end{aligned} \quad (42)$$

- *Step i (i = 3, \dots, n-1):* For  $V_i = V_{i-1} + (1/(n-i+2))(|z_i| - \delta_i)^{n-i+2} \eta_i(z_i)$ , we choose

$$\begin{aligned} \alpha_i &= -\left(c_i + \frac{5}{4}\right)(|z_i| - \delta_i)^{n-i+1} sg_i(z_i) \\ &\quad + \sum_{j=1}^{i-1} \left[ \frac{\partial \alpha_{i-1}}{\partial x_j} x_{j+1} + \frac{\partial \alpha_{i-1}}{\partial y_r^{(j-1)}} y_r^{(j)} \right] + \frac{\partial \alpha_{i-1}}{\partial \hat{\theta}} \Gamma \tau_i \end{aligned}$$

$$- \left( \hat{\theta}^T - \sum_{k=2}^{i-1} (|z_k| - \delta_k)^{n-i+1} \eta_k s g_k \frac{\partial \alpha_{k-1}}{\partial \hat{\theta}} \right) \cdot \left( \phi_i - \sum_{k=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_k} \phi_k \right) - (\delta_{i+1} + 1) s g_i(z_i), \quad (43)$$

$$\tau_i = \left( \phi_i - \sum_{k=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_k} \phi_k \right) (|z_i| - \delta_i)^{n+1-i} \eta_i s g_i(z_i) + \tau_{i-1}, \quad (44)$$

where  $c_i$  is a positive design parameter. The derivative of  $V_i$  is given by

$$\begin{aligned} \dot{V}_i \leq & - \sum_{i=1}^i c_i (|z_i| - \delta_i)^{2(n-i+1)} \eta_i + \tilde{\theta}^T (\tau_i - \Gamma^{-1} \dot{\hat{\theta}}) \\ & + \left( \sum_{k=2}^{i-1} (|z_k| - \delta_k)^{n-k+1} s g_k \frac{\partial \alpha_{k-1}}{\partial \hat{\theta}} \right) (\Gamma \tau_n - \dot{\hat{\theta}}) \\ & + (|z_i| - \delta_i)^{n-i+1} (|z_{i+1}| - \delta_{i+1} - 1) \eta_i. \end{aligned} \quad (45)$$

- *Step n:* Using (15), the derivative of  $z_n$  can be written as

$$\begin{aligned} \dot{z}_n = & v - \hat{W}^T \sigma(M^T u + V) M^T \tilde{W}_0^T \sigma(M_0, v, N_0) \\ & + \tilde{W}^T \sigma(M, u, V) M^T \hat{\omega}_{NN} + \phi_0(x(t)) \\ & + \theta^T \phi_n(x(t)) - y_r^{(n)} - \dot{\alpha}_{n-1} + d(t). \end{aligned} \quad (46)$$

The control law is designed as follows:

$$\begin{aligned} v = & - (c_n + 1) (|z_n| - \delta_n) s g_n(z_n) - \hat{\theta}^T \phi_n - \phi_0 - s g_n(z_n) \hat{\beta}^T Y \\ & + \sum_{j=1}^{n-1} \frac{\partial \alpha_{i-1}}{\partial x_j} x_{j+1} + \frac{\partial \alpha_{i-1}}{\partial \hat{\theta}} \dot{\hat{\theta}} \\ & + \sum_{j=1}^{n-1} \frac{\partial \alpha_{i-1}}{\partial y_r^{(j-1)}} y_r^{(j)} - \left( \phi_n - \sum_{k=1}^{n-1} \frac{\partial \alpha_{i-1}}{\partial x_k} \phi_k \right) \\ & \times \left( \hat{\theta}^T - \sum_{k=2}^{n-1} (|z_k| - \delta_k)^{n-k+1} s g_k \eta_k \cdot \frac{\partial \alpha_{k-1}}{\partial \hat{\theta}} \right), \end{aligned} \quad (47)$$

$$u(t) = v + \hat{\omega}_{NN}(v), \quad (48)$$

$$\hat{\omega}_{NN}(v) = \hat{W}_0^T \sigma(M_0, v, N_0). \quad (49)$$

The parameter update laws are designed as follows:

$$\dot{\hat{\theta}} = \Gamma \tau_n, \quad (50)$$

$$\dot{\hat{\beta}} = \Gamma_1 (|z_n| - \delta_n) \eta_n Y, \quad (51)$$

$$\dot{\hat{W}} = \Gamma_2 (|z_n| - \delta_n) \eta_n s g_n \sigma(M, u, N) M^T \hat{\omega}_{NN}, \quad (52)$$

$$\begin{aligned} \dot{\hat{W}}_0 = & - \Gamma_3 (|z_n| - \delta_n) \eta_n s g_n \hat{W}^T \sigma(M, u, N) \\ & \cdot M^T \sigma(M_0, v, N_0), \end{aligned} \quad (53)$$

where  $c_n$  is a positive parameter, and  $\Gamma, \Gamma_1, \Gamma_2, \Gamma_3$  are positive definite matrices.

**Theorem 2.** Consider the uncertain nonlinear system (1) satisfying Assumptions 1–2, the controller (48) and the parameter update laws (50)–(53). For bounded initial conditions, the following statements hold:

- The overall closed-loop system is semi-globally stable in the sense that all of the signals in the closed-loop system are bounded and the states  $x$  and the neural weight estimates  $\hat{W}, \hat{W}_0$  and parameter estimates  $\hat{\theta}, \hat{\beta}$  eventually converge to the compact set

$$\begin{aligned} \Omega_n \triangleq & \left\{ x, u, v, \hat{\theta}, \hat{\beta}, \hat{W}, \hat{W}_0 \mid |z_i| \leq \delta_i \right. \\ & + [(n-i+2)\mu]^{1/(n-i+2)}, \|\tilde{\theta}\|^2 \leq \frac{2\mu}{\lambda_{\min}(\Gamma^{-1})}, \\ & \|\tilde{\beta}\|^2 \leq \frac{2\mu}{\lambda_{\min}(\Gamma_1^{-1})}, \|\tilde{W}\|^2 \leq \frac{2\mu}{\lambda_{\min}(\Gamma_2^{-1})}, \\ & \left. \|\tilde{W}_0\|^2 \leq \frac{2\mu}{\lambda_{\min}(\Gamma_3^{-1})}, y_r^{(i-1)} \in \Omega_r \right\}, \end{aligned} \quad (54)$$

where  $\mu = V_n(0)$  is a positive constant.

- The tracking error approaches  $\delta_1$  asymptotically, i.e.,  $\lim_{t \rightarrow \infty} |y(t) - y_r(t)| = \delta_1$ . (55)

- The transient tracking error performance is given by

$$\begin{aligned} & \| |y(t) - y_r(t)| - \delta_1 \|_2 \\ & \leq c_1^{-1/2n} (\|\theta(0)\|_{\Gamma^{-1}}^2 + \|\beta(0)\|_{\Gamma_1^{-1}}^2 + \|W(0)\|_{\Gamma_2^{-1}}^2 \\ & \quad + \|W_0(0)\|_{\Gamma_3^{-1}}^2)^{1/2n}, \end{aligned} \quad (56)$$

with  $z_i(0) = \delta_i, i = 1, \dots, n$ .

**Proof.** We choose the Lyapunov function as follows:

$$\begin{aligned} V_n = & \sum_{i=1}^n \frac{1}{n-i+2} (|z_i| - \delta_i)^{n-i+2} \eta_i + \frac{1}{2} \tilde{\theta}^T \Gamma^{-1} \tilde{\theta} \\ & + \frac{1}{2} \tilde{W}^T \Gamma_2^{-1} \tilde{W} + \frac{1}{2} \tilde{W}_0^T \Gamma_3^{-1} \tilde{W}_0 + \frac{1}{2} \tilde{\beta}^T \Gamma_1^{-1} \tilde{\beta}. \end{aligned} \quad (57)$$

Since  $|d(t)| \leq \beta^T Y$ , by using adaptive laws (50)–(51), the derivative of  $V_n$  along (57) is given by

$$\begin{aligned} \dot{V}_n \leq & - \sum_{i=1}^n c_i (|z_i| - \delta_i)^{2(n-i+1)} \eta_i + \tilde{\theta}^T (\tau_n - \Gamma^{-1} \dot{\hat{\theta}}) \\ & + \left( \sum_{k=2}^{n-1} (|z_k| - \delta_k)^{n-k+1} s g_k \frac{\partial \alpha_{k-1}}{\partial \hat{\theta}} \right) (\Gamma \tau_n - \dot{\hat{\theta}}) \\ & + \tilde{W}^T [(|z_n| - \delta_n) \eta_n s g_n \sigma(M, u, N) M^T \hat{\omega}_{NN} \\ & - \Gamma_2^{-1} \dot{\hat{W}}] - \tilde{W}_0^T [\Gamma_3^{-1} \dot{\hat{W}}_0 + (|z_n| - \delta_n) \eta_n s g_n \end{aligned}$$

$$\begin{aligned} & \cdot \hat{W}^T \sigma(M, u, N) M^T \sigma(M_0, v, N_0) \\ & + \tilde{\beta}^T [(|z_n| - \delta_n) \eta_n Y - \Gamma_1^{-1} \hat{\beta}] \\ = & - \sum_{i=1}^n c_i (|z_i| - \delta_i)^{2(n-i+1)} \eta_i. \end{aligned} \tag{58}$$

The boundedness of  $V_n$  can now be established. Integrating both sides of (58) gives

$$V_n(t) + \int_0^t \sum_{i=1}^n c_i (|z_i| - \delta_i)^{2(n-i+1)} \eta_i d\tau \leq V_n(0). \tag{59}$$

It implies  $V_n(t)$  is bounded. The boundedness of  $V_n$  further implies that  $z_i, i = 1, \dots, n, \tilde{\theta}, \tilde{W}, \tilde{W}_0, \tilde{\beta}$  are bounded. From (47) and (48), the boundedness of  $v(t)$  and  $u(t)$  is obtained.

Let  $\mu = V_n(0)$ . Clearly from (59), we can obtain that

$$|z_i| \leq \delta_i + [(n - i + 2)\mu]^{1/(n-i+2)}, \tag{60}$$

$$\begin{aligned} \|\tilde{\theta}\|^2 & \leq \frac{2\mu}{\lambda_{\min}(\Gamma^{-1})}, \quad \|\tilde{\beta}\|^2 \leq \frac{2\mu}{\lambda_{\min}(\Gamma_1^{-1})}, \\ \|\tilde{W}\|^2 & \leq \frac{2\mu}{\lambda_{\min}(\Gamma_2^{-1})}, \quad \|\tilde{W}_0\|^2 \leq \frac{2\mu}{\lambda_{\min}(\Gamma_3^{-1})}. \end{aligned} \tag{61}$$

Therefore, we can conclude that all signals remain in the compact set  $\Omega_n$  (54), where the size of  $\mu$  can be adjusted by appropriately choosing the design parameters. Adaptive neural and fuzzy control schemes were proposed in [4,10,13], where system states are guaranteed to stay in a compact set.

From the Barbalat’s lemma, we can conclude that for  $1 \leq i \leq n, \lim_{t \rightarrow \infty} (|z_i| - \delta_i)^{n-i+1} f_i = 0$ , which implies that  $\lim_{t \rightarrow \infty} \hat{\theta} = 0, \lim_{t \rightarrow \infty} \hat{W} = 0, \lim_{t \rightarrow \infty} \hat{W}_0 = 0$ , and  $\lim_{t \rightarrow \infty} \hat{\beta} = 0$ , and in particular,  $y - y_r$  converges to  $[-\delta_1, \delta_1]$ .

From (59), we have

$$\| |z_1| - \delta_1 \|_2^{2n} = \int_0^\infty (|z_1| - \delta_1)^{2n} d\tau \leq \frac{1}{c_1} V_n(0). \tag{62}$$

Be setting  $z_i(0) = \delta_i, i = 1, \dots, n$ , the bound is given by

$$\begin{aligned} \| |z_1| - \delta_1 \|_2 & \leq c_1^{-1/2n} (\|\theta(0)\|_{\Gamma^{-1}}^2 + \|\beta(0)\|_{\Gamma_1^{-1}}^2 \\ & + \|W(0)\|_{\Gamma_2^{-1}}^2 + \|W_0(0)\|_{\Gamma_3^{-1}}^2)^{1/2n}. \quad \square \end{aligned} \tag{63}$$

**Remark 5.** Since the NN approximation (5) is only guaranteed within a compact set, the stability result proposed in this work is semi-global in the sense that, for any compact set, there exists a controller with NN approximation such that all the closed-loop signals are bounded when the initial states are within this compact set. In practical applications, the number of nodes usually cannot be chosen too large due to the possible computation problem. This implies that the SLFN approximation capability is limited, and some constraints on  $\Omega_n$  are necessary to guarantee the NN approximation.

**Remark 6.** From Theorem 2, the following conclusions can be obtained:

- The transient performance depends on the initial estimate errors  $\tilde{\theta}(0), \tilde{W}(0), \tilde{W}_0(0), \tilde{\beta}(0)$  and explicit design parameters. The closer the initial estimates  $\hat{\theta}(0), \hat{W}(0), \hat{W}_0(0), \hat{\beta}(0)$  to the true values  $\theta, W, W_0$  and  $\beta$ , the better the transient performance is.
- The bound for  $\|y(t) - y_r(t)\|_2$  is an explicit function of design parameters and thus is computable. We can reduce the effects of the initial error estimates on the transient performance by increasing the adaptation gains  $\Gamma, \Gamma_1, \Gamma_2, \Gamma_3$ .

### 5. Simulation studies

In this section, we illustrate the above methodology on a system which is described by

$$\dot{x} = a \frac{1 - e^{-x(t)}}{1 + e^{-x(t)}} + w + d(t), \tag{64}$$

$$w = N(u)$$

$$= \begin{cases} m_r(u(t) - b_r), & u(t) \geq b_r, \\ 0, & b_l < v(t) < b_r, m, y = x(t), \\ m_l(u(t) - b_l), & u(t) \leq b_l, \end{cases} \tag{65}$$

where  $a$  is an uncertain parameter,  $d(t)$  is unknown disturbance and  $w$  represents the output of the dead-zone nonlinearity. The actual parameter value is  $a = 1$  and  $d(t) = 0.2 \sin(t)$ . The parameters of the dead-zone are  $b_r = 0.5, b_l = -0.6$  and  $m_r = 1, m_l = 1.2$ . The objective is to control the system state  $y(t)$  to follow a desired trajectory  $y_r(t) = 2.5 \sin(t)$ .

The adaptive NN controller  $v$  is chosen according to (47) as follows:

$$v = -(c_1 + 1)(|z_1| - \delta_1) sg_1(z_1) - \hat{\theta} \phi_1 - sg_1(z_1) \hat{\beta}^T Y + \dot{y}_r, \tag{66}$$

where  $z_1 = x_1 - y_r, \theta = a, \phi_1 = (1 - e^{-x(t)})/(1 + e^{-x(t)})$ . The control law is given by (48). The adaptive laws for the neural networks weights  $\hat{W}$  and  $\hat{W}_0$  are given by (52) and (53), respectively. The adaptive laws for  $\hat{\theta}$  and  $\hat{\beta}$  are described by (50) and (51).

The two NN approximators are given in (12) and (13). The NN I contains  $L = 10$  hidden-layer nodes with sigmoidal function. The first layer weights  $M$  are uniformly randomly distributed between  $-1$  and  $+1$ . The threshold weights  $N$  for the first layer are uniformly randomly distributed between  $-5$  and  $+5$ . The threshold weights represent the bias in sigmoidal functions positions. They should cover the range of the dead-zone. As in [11], it is recommended that this range is large enough so it covers the dead-zone width. The second-layer weights  $\hat{W}$  are uniformly randomly initialized in the region  $[-10, +10]$ . The NN II contains  $L = 10$  hidden-layer nodes with sigmoidal functions. The first-layer weights  $M_0$  are

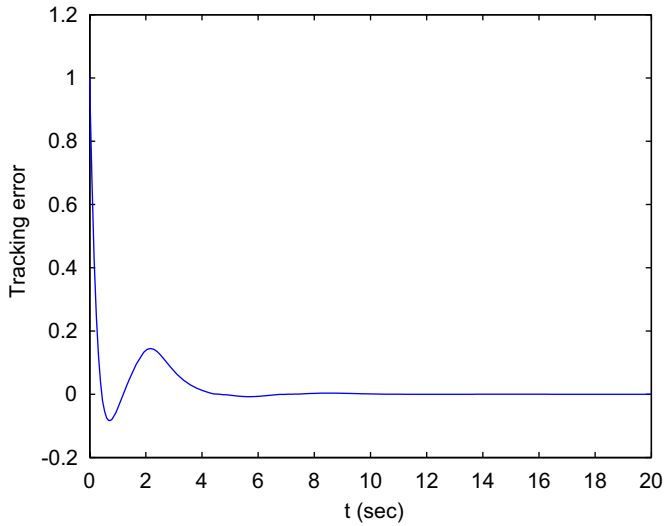
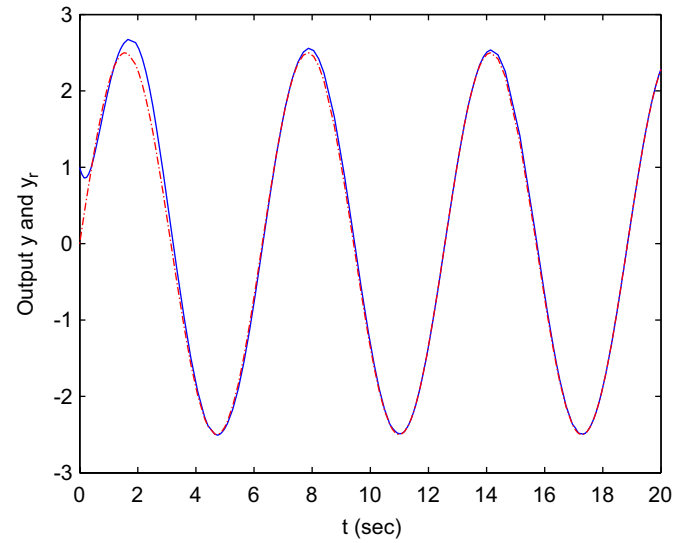
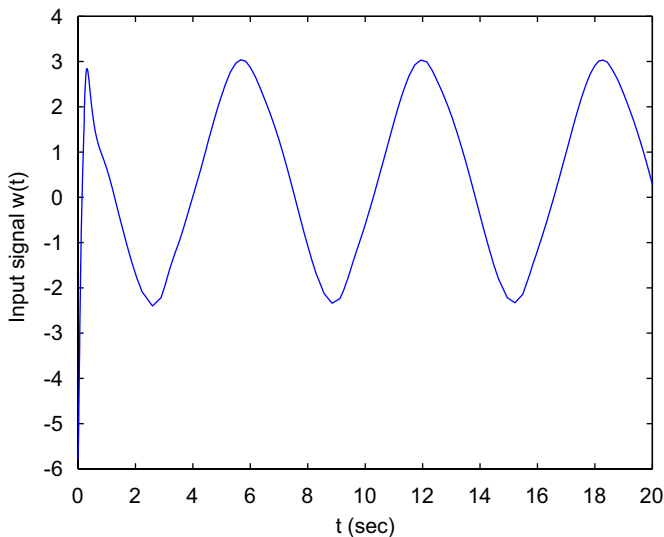


Fig. 3. Tracking error.

Fig. 5. Output  $y(t)$  (solid line) and trajectory  $y_r(t)$  (dashed line).Fig. 4. Control signal  $u(t)$ .

uniformly randomly distributed between  $-1$  and  $+1$  as in NN I and the thresholds weights  $N_0$  are uniformly randomly distributed between  $-5$  and  $+5$ . The second-layer weights  $\hat{W}_0$  are initialized at zero. The design parameters are chosen as  $c_1 = 0.8$ ,  $\delta_1 = 0.1$ ,  $\Gamma = 0.1$ ,  $\Gamma_1 = \text{diag}\{0.2\}$ ,  $\Gamma_2 = \text{diag}\{2.0\}$ ,  $\Gamma_3 = \text{diag}\{1.0\}$ . The initial estimate  $\hat{\theta}(0) = 0.5$  and the initial state  $x(0) = 1.0$ .

The simulation results are shown in Figs. 3–5. Figs. 3–5 show the system tracking error, the input and the output and trajectory. Clearly, all the results verify our theoretical findings and demonstrate the effectiveness of the proposed control scheme.

## 6. Conclusions

In this paper, a new adaptive control architecture is proposed for a class of nonlinear uncertain systems

containing nonlinearities in the actuator device. To address such a challenge, neural networks (NNs) will be adopted to approximate nonlinearities in the plant model and the controller is constructed based on NNs. The controller is designed by using backstepping technique incorporated with certain well defined sign functions and NNs approximations, where the NNs are adjusted by an adaptive law. The proposed adaptive control law not only can guarantee stability, but also achieve excellent transient performance. It turns out that the tracking error is an explicit function of design parameters and thus the proposed scheme allows designers to achieve closed-loop behavior by tuning design parameters in an explicit way.

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